$$= c^{(k+1)-1} S(1) + \sum_{i=2}^{k+1} c^{(k+1)-i} g(i)$$

$P(k+1)$.

$\therefore$ By the 1st principle of induction

$P(n)\ \forall n \geq 1$

$T(1) = 1 \leftarrow c=1$

$T(n) = T(n-1) + 3 \quad n \geq 2$

$\uparrow c=1 \qquad \uparrow g(n)=3$

$T(n) =$

$$1^{n-1} T(1) + \sum_{i=2}^{n} 1^{n-i} g(i)$$

$$= T(1) + \sum_{i=2}^{n} 3$$

$$= 1 + (n-1)3 = \boxed{3n-2}$$

Assume a divide & conquer recurrence

like

$$S(1) = a$$
$$S(n) = c\,S(n/2) + g(n)$$

Assume $n = 2^m$

$$S(2^0) = a$$
$$S(2^m) = c\,S(2^{m-1}) + g(2^m)$$

Let $T(m) = S(2^m)$

$$T(0) = a$$
$$T(m) = c\,T(m-1) + g(2^m)$$
$$= c^{m-1}\,T(1) + \sum_{i=2}^{m} c^{m-i}\, g(2^i)$$

Substitute back in for S & n:

$$m = \log_2 n \iff 2^m = n$$

$$S(n) = c^{\log_2 n - 1}\, S(2) + \sum_{i=2}^{\log_2 n} c^{\log_2 n - i}\, g(2^i)$$
$$= c^{\log_2 n - 1}\left[c\,S(1) + g(2)\right] + \sum_{i=2}^{\log_2 n} c^{\log_2 n - i}\, g(2^i)$$

$$\boxed{S(n) = c^{\log_2 n} S(1) + \sum_{i=1}^{\log_2 n} c^{\log_2 n - i} g(2^i)}$$

$a = 3$

$c = 1$

$g(n) = n$ identity function

$$S(n) = 3 + \sum_{i=1}^{\log_2 n} 2^i$$

$$= 3 + \underbrace{2^1 + 2^2 + 2^3 + \dots + 2^{\log_2 n}}_{\text{geometric series}}$$

$$= 2 + \underbrace{1 + 2^1 + 2^2 + \dots + 2^{\log_2 n}}$$

$$= 2 + \frac{1 - 2^{\log_2 n + 1}}{1 - 2}$$

$$= 2 + 2^{\log_2 n + 1} - 1$$

$$= 2 + 2 \cdot 2^{\log_2 n} - 1$$

$$= 2 + 2 \cdot n - 1$$

$$= \boxed{2n + 1}$$

$F(n) = F(n-1) + F(n-2)$

$F(1) = 1$

$F(2) = 1$

$A(n)$ - sums required for the the computation of the n^{th} Fibonacci.

$A(1) = 0$

$A(2) = 0$

$A(3) = 1$

$A(4) = 2$

$A(5) = 4$

$F(3) = F(2) + F(1) \Rightarrow 1$

$F(4) = F(3) + F(2) \Rightarrow 2$

$$\boxed{A(n) = A(n-1) + A(n-2) + 1}$$

2^{nd} order linear recurrence relation with constant coefficients (non-homogeneous)