Let $n \in \mathbb{Z}$. Prove:

If n^2 is odd, then n must be odd

n^2 is odd $\rightarrow$ n is odd

Contrapositive:

$(n \text{ is odd})' \rightarrow (n^2 \text{ is odd})'$

n is even $\rightarrow$ n^2 is even

Let n be even: $\exists k \in \mathbb{Z}$ /

$n = 2k$ (defn. of evenness)

Then

$(n^2) = (2k) \cdot (2k) = 2(2k^2)$ commutativity associativity

$2k^2$ is an integer by closure of integers under multiplication.

So n^2 is even by the defn. of evenness.

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$T(1) = 3$

$T(2) = \cancel{T}(1) + 5 * T(1) - 1$

$T(3) = T(2) + 5 \cdot T(2) - 6$

$T(4) = T(3) + 5(T(3)) - 6^2$

$T(n) = 6(T(n-1)) - 6^{n-2}$ $\quad g(n) = -6^{n-2}$

$6^{n-1} \cdot 3 + \sum_{i=2}^{n} 6^{n-i}(-6^{i-2})$

$= 3 \cdot 6^{n-1}$

$= \frac{1}{2} 6^n - \sum_{i=2}^{n} 6^{n-2}$

$= \frac{1}{2} 6^n - (n-1) 6^{n-2}$

$= \left(\frac{1}{2} - \frac{n-1}{36}\right) 6^n$

$n = 19$

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$S(1) = 1$

$S(n) = nS(n-1) + n!$ for $n \geq 2$

$S(2) = 2 \cdot 1 + 2! = 2! + 2! = 2 \cdot 2!$

$S(3) = 3(2! + 2!) + 3!$

$S(3) = 3! + 3! + 3! = 3 \cdot 3!$

$S(4) = 4(3! + 3! + 3!) + 4! = 4 \cdot 4!$

$P(n): S(n) = n \cdot n!$ Theorem!

Prove it.

Anchor: $S(1) = 1 \cdot 1! = 1$ ✓ $P(1)$

Inductive: Assume $P(k): S(k) = k \cdot k!$

Consider $P(k+1): S(k+1) = (k+1)(k+1)!$

$$S(k+1) = (k+1)S(k) + (k+1)!$$
$$= (k+1)k \cdot k! + (k+1)!$$
$$= k\underbrace{(k+1)k!}_{} + (k+1)!$$
$$= k(k+1)! + 1 \cdot (k+1)!$$
$$= (k+1)(k+1)!$$

✓

Problem 4: (10 pts) Decide if the following wff is valid, and either give an interpretation in which it is false, or prove it:

$$(\exists x)\,(P(x) \to Q(x)) \wedge (\forall y)\,(Q(y) \to R(y)) \to ((\forall x)P(x) \to (\exists x)R(x))$$

1. $(\exists x)\,(P(x) \to Q(x))$ hyp
2. $(\forall y)\,(Q(y) \to R(y))$ hyp
3. $(\forall x)\,P(x)$ ded. method

$$\left[(\exists x)(P(x) \to Q(x)) \wedge (\forall y)(Q(y) \to R(y)) \wedge (\forall x)P(x) \to (\exists x)R(x) \right]$$

4. $P(x) \to Q(x)$ 1, ei.
5. $Q(x) \to R(x)$ 2, ui
6. $P(x)$ 3, ui
7. $Q(x)$ 4,6 mp.
8. $R(x)$ 5,7 mp
9. $(\exists x)\,R(x)$ 8, e.g.

6. $P(x) \to R(x)$ 4,5 hs
7. $P(x)$
8. $R(x)$ 6,7, mp
9. $(\exists x)\,R(x)$

---

1. $P \to Q$
2. Q'
3. P' 1,2, mt

Problem 6: (10 pts) Use induction to prove the formula for the sum of the first n terms of a geometric sequence, for $n \geq 1$:

$$P(n): \quad a + ar + ar^2 + \ldots + ar^{n-1} = a\frac{1-r^n}{1-r}$$

Anchor:

$$P(1): \quad a = a \cdot \frac{1-r}{1-r} = a \quad \checkmark$$

Inductive:

Assume $P(k)$: $a + ar + \ldots + ar^{k-1} = a\frac{1-r^k}{1-r}$

Consider $P(k+1)$: $a + ar + \ldots + ar^{k-1} + ar^{(k+1)-1} = a\frac{1-r^{(k+1)}}{1-r}$

$$\underbrace{a + ar + \ldots + ar^{k-1}}_{\text{by the inductive hypothesis}} + ar^{(k+1)-1} =$$

$$a\frac{1-r^k}{1-r} + ar^k =$$

$$a\left[\frac{1-r^k}{1-r} + r^k\frac{1-r}{1-r}\right] =$$

$$a\left[\frac{1-r^k+r^k-r^{k+1}}{1-r}\right] =$$

$$a\left[\frac{1-r^{k+1}}{1-r}\right] \quad \checkmark$$

$\therefore$ $P(n)$ $\forall n \in \mathbb{N}$ by the 1st principle of mathematical induction.