MAT129 Test 2 (Spring 2018): Derivatives and their uses

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Note**: you must skip **one** of the problems marked "skippable". Write skip across it, so I'll easily know which one. **Good luck!**

Problem 1 (skippable, 10 pts): Demonstrate your differentiation prowess (simplify where possible):

a. Quotient rule: $f(x) = \dfrac{\cos(x)}{x^2 - 3}$

b. Chain rule: $g(x) = \cos(x^2 - 3)$

c. Chain rule: $p(x) = (\cos(x))^2 - 3$

d. Product rule: $q(x) = \cos(x)(x^2 - 3)$

Problem 2(10 pts): Assume that f and g are differentiable at x, and that $p(x) = f(x) + g(x)$.

a. (7 pts) Use the limit definition of the derivative to demonstrate the sum rule – that is, that

$$p'(x) = f'(x) + g'(x)$$

b. (3 pts) Suppose that f and g above are both differentiable on $[a, b]$, where a and b are real numbers. What does the Extreme Value Theorem guarantee about the function p?

Problem 3(10 pts): Related Rates: A ferris wheel spinning at 90 mph flies off its stand and hits an ice cream truck traveling 30^o north-by-northwest at 12 ft/s, as an extension ladder slides off the truck into the southbound lane at half that speed. The driver of the truck (a mathematician) dies while considering the following related rates problem:

An oil slick is spreading out in a circle from an oil drilling platform, which is leaking oil into a calm sea. The slick is one cm thick. The oil is spilling at the rate of 1000 liters[1] per second.

a. (2 pts) What is the volume of the slick (a cylinder of circular cross section of radius r and height h)? (Draw it. I'll sell the answer to you, if you can't figure it out.)

b. (4 pts) How fast is the slick's radius increasing when the slick is 25 meters in radius?

c. (4 pts) How fast is the slick's area changing at the very same moment?

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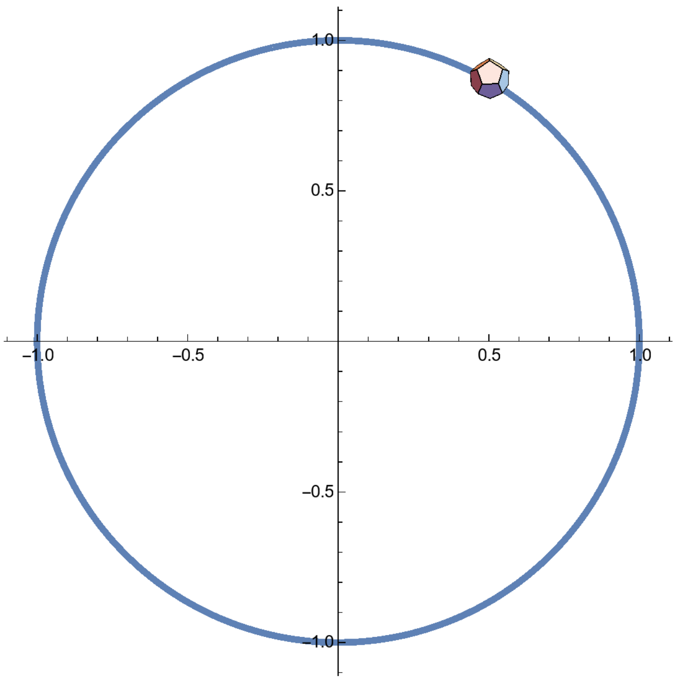
[1] A liter is 1000 cm^3.

Problem 4 (skippable, 10 pts): Consider the unit circle ($x^2 + y^2 = 1$) at the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

a. (6 pts) Demonstrate implicit differentiation to find the equation of the tangent line at that point. Sketch in your tangent line.



(4 pts) Now do the same problem **explicitly**:

a. Solve the equation of the unit circle for y as the appropriate **explicit** function of x.

b. Differentiate your expression $y(x)$ to obtain the slope of the tangent line at $x = \frac{1}{2}$.

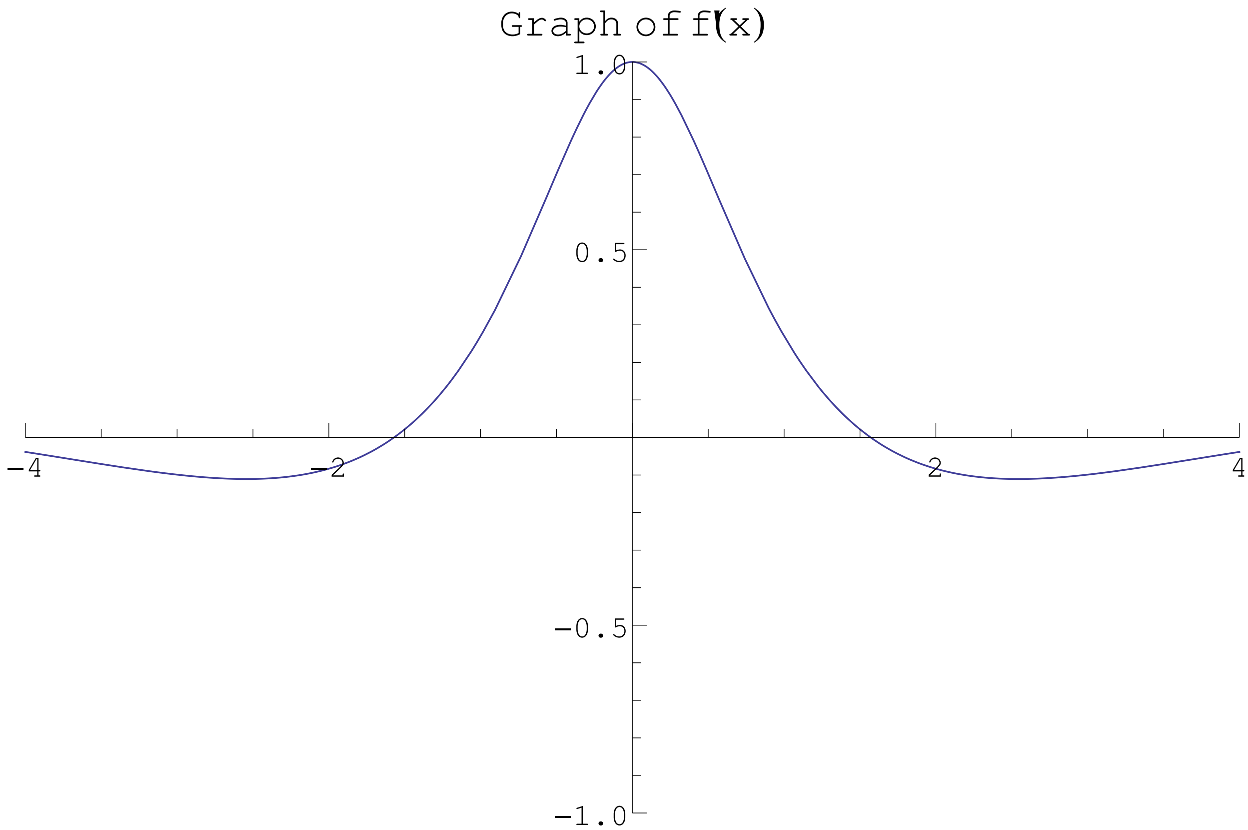
Problem 5 (skippable, 10 pts): Discover the precise end behavior (asymptotic behavior) of each of the following functions, as $x \to \infty$:

a. (4 pts) $f(x) = \dfrac{x^2 - 3x + 2}{x + 1}$

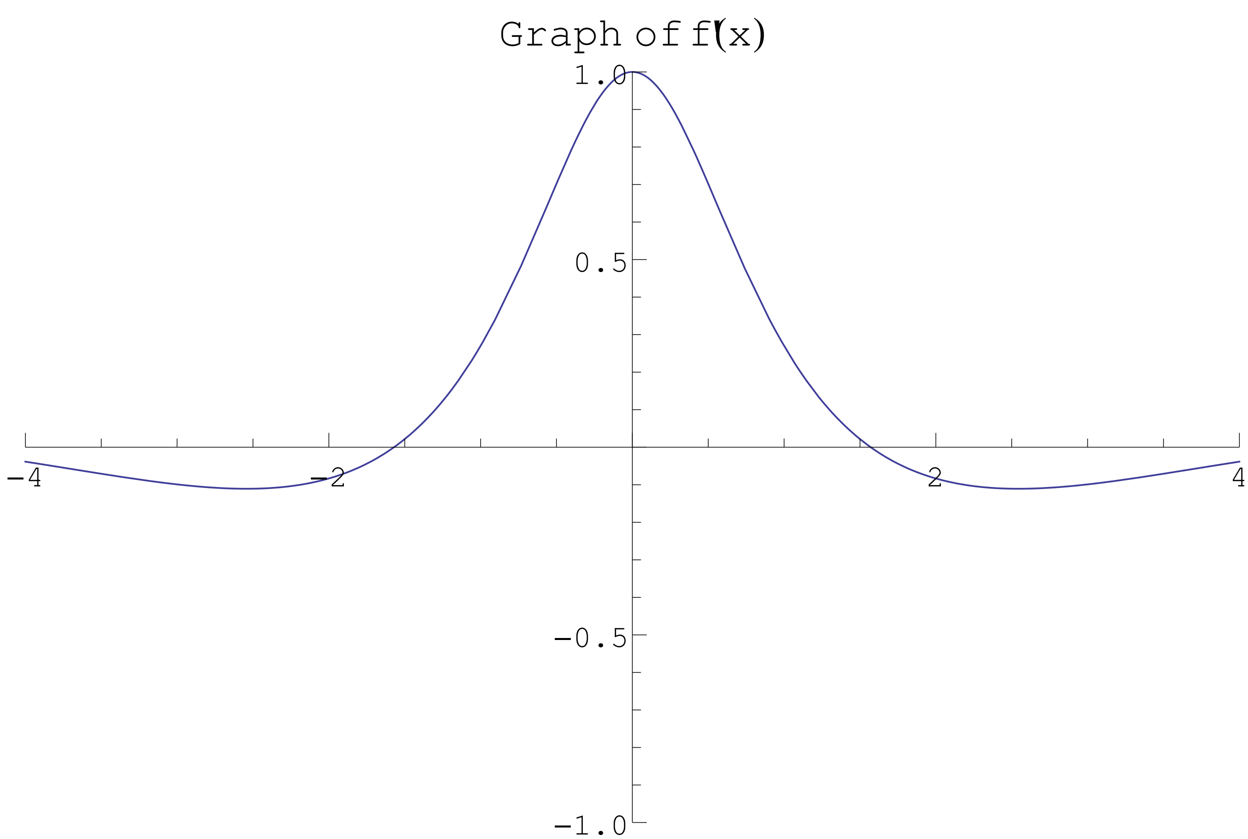
b. (4 pts) $g(x) = \dfrac{2x^2 - 3x + 2}{\sqrt[3]{x^6 + 1}}$

c. (2 pts) $h(x) = \sqrt{x^2 + 2x - 2}$

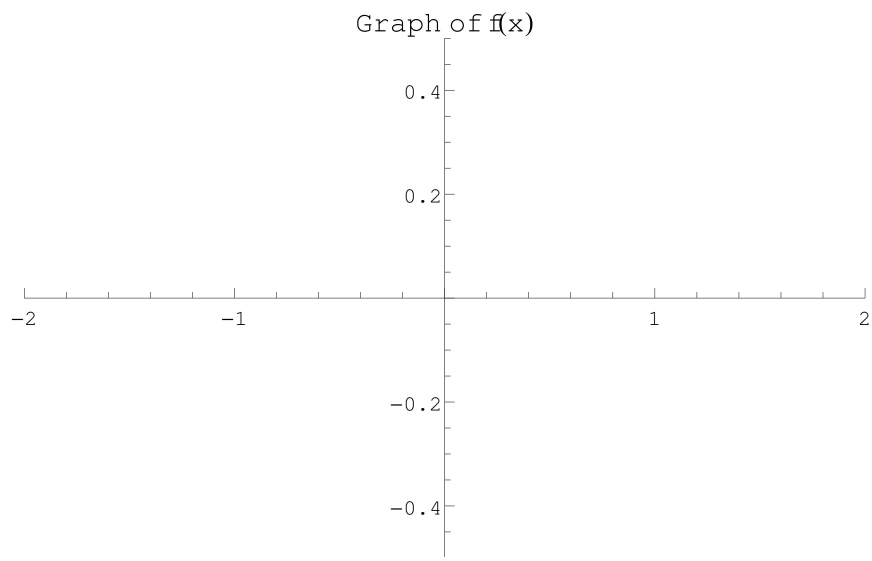
Problem 6 (skippable, 10 pts): What follows is the graph of the derivative of $f(x)$, $f'(x)$. Graph the second derivative, $f''(x)$, on the same coordinate system with $f'(x)$:



Use information you can glean from $f'(x)$ and $f''(x)$ to **carefully** sketch the graph of the function f (assume that $f(0) = 0$).



Problem 7 (10 pts): Do a study of the function $\frac{x}{x^2+1}$, culminating in a careful plot of this function on the axes below.



f	
f'	
f''	