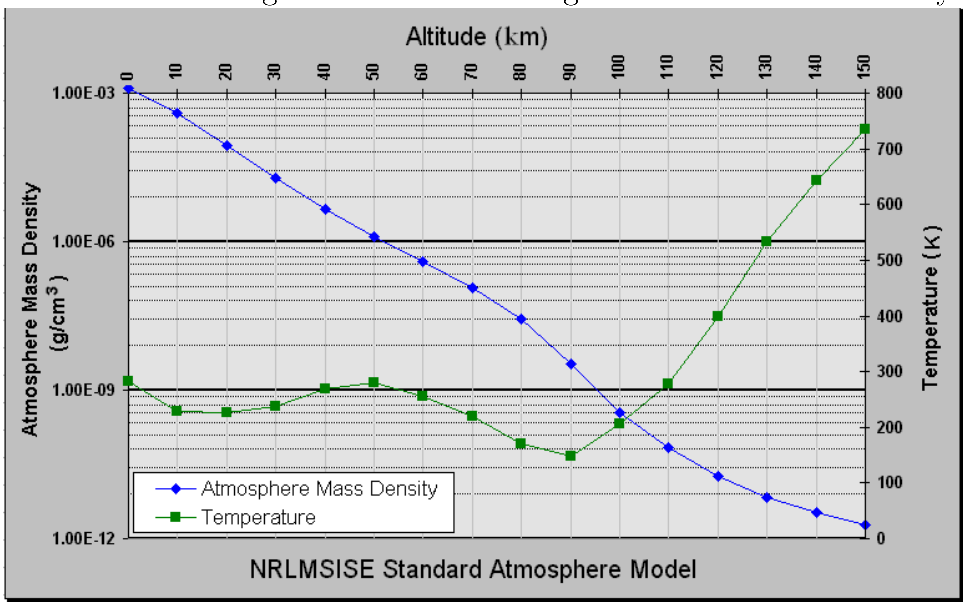
Section 6.3 (and 6.4, really!): Volumes

How massive is our atmosphere? Scientists have measured the average mass density of this cushion over the Earth, as shown in this figure[1]:

Figure 1: According to the National Center for Atmospheric Research, "The total mean mass of the atmosphere is 5.1480 $\times 10^{18}$ kg...." Notice the strange scale for the mass density.



There is a shell of air around the Earth (whose radius is 6,360 km), and the mass density decreases with height until its mass density is zero (at about 160 km, let's say, based on the data in the figure).

a. How might you set up the integral to evaluate the mass of the atmosphere? Discuss.

b. In this case, disks and washers would be a royal pain. We can do it, but you might be sorry!;) Just calculating the volume occupied by the atmosphere is hard enough – let's try to just set up the integral. Draw the earth as a circle at the center of your axis system, and add the atmosphere as a ring around it. If you draw it to scale, the ring would be very thin – so don't!

[1] `http://upload.wikimedia.org/wikipedia/commons/d/de/Atmosphere_model.png`

c. The surface area of a sphere is $4\pi r^2$. How can we use this (and the data in the figure) to quickly estimate the mass? (Hint: use averages! Try $4.524 * 10^{-5} gm/cm^3$ for an average mass density. I get $3.77265 * 10^{18} kg$.)

d. Now set up an integral made up of "spherical shells" (like the layers of an onion) to calculate the mass of the atmosphere. Pay careful attention to units! Here's a rough model for the data in the graphic:

$$\rho(x) = 10^{-3(1+x/50)}$$

(I get $3.68762 * 10^{18} kg$ for an answer.)