Section Summary: 1.1

a. Definitions

- **function**: A **function** is a rule that assigns to each element x in a set D **exactly one element** (called $f(x)$) in a set E.

  Functions are represented in (at least) four different ways: by graphs, tables, formulas, or by verbal description.

- **graph of a function**: the **graph of a function** is the set of ordered pairs $\{(x, f(x)) \mid x \in D\}$. D is called the **domain**, and the set of all values $f(x)$ for $x \in D$ is called the **range** of f: $\{f(x) \mid x \in D\}$.

- **Symmetry**:

  i. **even** function: if a function f satisfies $f(-x) = f(x)$ for every number in its domain, the f is called an **even** function.

  ii. **odd** function: if a function f satisfies $f(-x) = -f(x)$ for every number in its domain, the f is called an **odd** function.

  The graphs of these functions possess symmetry (even functions have reflective symmetry about the line $x = 0$, while odd functions have rotational symmetry about the origin). An example of an even function is $f(x) = x^2$; an example odd function is $f(x) = x^3$ (note the powers!).

- The **absolute value** function:

$$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$$

  Notice that $|a| \geq 0$ for all values of a.

  This is an example of a **piecewise** defined function – it has two different formulas, on two parts of the x-axis.

- **increasing** function: A function f is **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \in I$$

- **decreasing** function: A function f is **decreasing** on an interval I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 > x_2 \in I$$

b. Theorems

The **Vertical Line Test**: A curve in the xy-plane is the graph of a function f of x if and only if no vertical line intersects the curve more than once.

c. Summary

This section introduces several important properties of functions: how to represent them, the uniqueness that leads to the vertical line test, symmetry, and properties such as increasing/decreasing, and piecewise definition.

These should be review, from pre-calc, but sometimes it's been awhile....

Some good graphs/examples to consider include Figure 1, p. 10 (not all functions are nice!); Figures 9 and 10, p. 13 (we are often interested in fitting a curve to a set of points); Figure 11, p. 14 (how would the function change if the water running out were only a drip?); Figure 16, p. 16 shows the absolute value function in all its glory (its graph possesses what symmetry?); Figure 21 has a gallery showing symmetry (and lack of symmetry).