Section Summary: 1.2

a. Definitions

- **mathematical model**: A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real-world phenomoenon such as the size of a population, the demand for a product, etc.
- **linear models**: straight lines
- **polynomials models**: $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$
  E.g. linear (degree $n = 1$), quadratic (degree $n = 2$), cubic (degree $n = 3$), quartic (degree $n = 4$), quintic (degree $n = 5$), etc.
- **power functions**: $f(x) = x^a$, where $a \in \mathbb{R}$.
  E.g. square root functions, x^{-1}.
- **rational functions**: $f(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials.
  E.g. mobius functions $\dfrac{ax+b}{cx+d}$ (whose graphs are hyperbolas).
- **Trigonometric functions**: sine, cosine, tangent are the most important.
- **Exponential functions**: $f(x) = a^x$, where $a > 0$.
  E.g. e^x, 2^x.
  Logarithmic functions: inverses of the exponential functions.
  E.g. $\ln(x)$, $\ln_2(x)$.

b. Important properties

$$-1 \leq \sin(x) \leq 1$$
$$-1 \leq \cos(x) \leq 1$$

$$\sin(x + 2\pi) = \sin(x)$$
$$\cos(x + 2\pi) = \cos(x)$$

Since we have these two properties, it turns out that we only really need one of these trig functions:

$$\cos(x) = \sin(x + \frac{\pi}{2})$$
$$\sin(x) = \cos(x - \frac{\pi}{2})$$

Nonetheless, we like to have them both around. Also

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

Notice that sine and tangent are odd, and that cosine is even.

c. **Summary**

This section presents us with many of the most important families of functions, members of what I call "the function zoo."

These should be review, from pre-calc, but sometimes it's been awhile....

Some good graphs/examples to consider include the illustration of curve-fitting (again, one of the most important things we do with calculus), Example 2, p. 25. The fit is not very good, however, since you can see a quadratic/parabolic trend in the data which the straight line fails to catch (Figure 6, p. 26).

Figures 7 and 8 (p. 27) show typical polynomial graphs; Figures 14 and 16 (p. 30) show that rational functions can give rise to vertical asymptotes, which is important. You need to memorize Figures 18 and 19 (p. 31 and 32, the most important trig functions). And you should know what typical exponentials and logs look like (Figures 20 and 21, p. 32).