Section Summary: 3.7

a. Theorems

First derivative test for absolute extreme values Suppose that c is a critical number of a continuous function f defined on an interval I.

- If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum of f on I.
- If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum of f on I.

b. Properties/Tricks/Hints/Etc.

Steps in solving optimization problems:

- Understand the problem - read carefully: what is the unknown? What are the given quantities? What are the given conditions? What do we seek?
- Draw an appropriate, complete diagram. (This is the most important step!)
- Introduce suitable notation. Use symbols that represent the quantities of interest (e.g. h for height, v for velocity), rather than just x or y.
- Express the dependent variable as an equation in terms of the independent variable(s).
- If there is more than one independent variable, use the relationships among the variables and conditions to solve for the others in terms of a single variable, if possible.
- Find the desired absolute minimum or maximum.

c. Summary

This section is the heart of calculus for me. There is something wonderful about being able to determine the best shape of a can to minimize the use of materials, or to discover that if you want Fido to have the biggest pen area given a rectangular fence, then it should be square. This is marvellous stuff.

Take to heart the recommendations for solving these optimization problems (in particular, **draw a picture**). Word problems are notoriously difficult, so turn them into picture problems to make them easier to solve.