# Section Summary: 5.2

## a. Definitions

- **cross-section**: an intersection of a solid with a plane, which gives rise to an area. We then multiply these areas times a (really thin) thickness, to give rise to a volume.
- **cylinder**: an object with a constant cross-section, and a given thickness, or height. For example, what we ordinarily call a "right circular cylinder" has circular cross-sections. For these objects the volume is simple to calculate: $V = Ah$, where $A$ is the area of the cross-section, and $h$ is the height of the stack.
- **volume**: Let $S$ be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of $S$ in the plane $P_x$, through $x$ and perpendicular to the $x$-axis, is $A(x)$, where $A$ is a continuous function, then the volume of $S$ is

  $$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)dx$$

  Actually, I prefer to start by thinking of every volume integral as

  $$V = \int_a^b dV(x)$$

  Then we figure out how to write an infinitesimal chunk of volume, $dV$, as a function of $x$. Often we'll break it down as a slice, with surface area $A(x)$ and thickness $dx$:

  $$V = \int_a^b dV(x) = \int_a^b A(x)dx$$

- **solids of revolution**: solids obtained by revolving a region about a line.

## b. Properties/Tricks/Hints/Etc.

For a solid of revolution, the trick is to compute the radius $r(x)$ (or, sometimes $r(y)$, if it's convenient to do the marching along the $y$-axis), and then "rove over" the appropriate axis.

We often end up working with "disks" or "washers" (which are really the difference of two disks).

## c. Summary

This is a step up from computing areas as sums of tiny rectangles of height $f(x)$ and width $dx$ (area $f(x)dx$): we're now computing volumes, by adding up little slabs of volume in the form of slices of area $A(x)$ times thickness $dx$ (volume $A(x)dx$). The conceptual idea is the same, but we're in one higher dimension.