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11 Two Models of Cultural Stability

Anthropology needs mathematics, not because mathematics is glamorous these days, but because mathematics can help anthropologists solve the kinds of problems anthropologists want to solve.

Paul Kay

I. INTRODUCTION

Communities of people cannot long survive unless the basic needs of the inhabitants are met. In a rudimentary "society" each individual might take care only of his own requirements. He would find food, gather and prepare it, build his own shelter, and provide his own entertainment, medical care, and transportation. In most societies, however, people are dependent on one another for various goods and services. There is a division of labor among the residents. One person or group of persons specializes in constructing houses while another harvests the crops. Certain members debate and modify the laws while others insure that violators are apprehended and punished.

Furthermore, the obligations and the privileges of a single member of the society are different at different stages of the person's life. The social and economic contributions to the community of a 7-year-old, for example, vary from that of a 47-year-old. These in turn are not the same as those of a person of age 77.

Other factors besides age are often important in determining what is expected of an individual or what he is allowed to do. The person's sex, race, religion, and perhaps even height and weight, can control what occupations he will pursue and the extent of his power or influence in the society. The continued cultural viability of a community may depend quite crucially on the factors that are used to structure the division of privilege and responsibility among the members.

II. THE GALLA SYSTEM

In this chapter, we will examine a system for the division of labor that has been used by some of the Galla tribes in Ethiopia. We shall consider only a simplified version of the actual system, so that we may focus on some important questions. In the Galla system, the critical functions of the tribe are structured through five age grades, called the Dabella, Folle, Kondala, Luba, and Yuba.

Each male in the society moves through the age-grade system, spending a period of eight consecutive years in each grade. Since there are five grades, it takes an individual 40 years to pass through the system. The key feature of this age-grade system is that a man enters the lowest grade at the moment his father retires from the highest grade. In other words, a son enters the system exactly 40 years after his father enters.

To illustrate this scheme, suppose your father enters the lowest grade when he is 13 years old and that you are born when he is 30 years old. Then your father retires from the system when he is 53 (40 + 13) years old. At that time, you will enter the system. Your age will be $53 - 30 = 23$.

To continue this example, suppose that you have two sons, one who is born when you are 35 and the other when you are 45. You will leave the system at age 63. Your two sons will enter the system at the same time, although their ages will be different. The elder will be 28 years old and the younger will be

18 years old. They will move through the system together, entering the successive grades at the same time, and retiring in the same year.

The calendar years of entrance into the Galla system of all the male descendents of a man is then determined once we know the year the man himself entered. If a man enters the lowest grade in the year 1900, all his sons enter in the year 1940, even if the man dies before the date of his retirement. It is also possible that a son may enter the system before he is born! To see how this may happen, suppose that the man who entered the system in 1900 was very young. Then it is quite conceivable that he has a son who is born in 1950. In such a case, it is assumed that the son entered the lowest (Dabella) grade in 1940. At his birth then, the son is considered to be a member of the Folle grade and will advance to the Konda grade in 1956. This son would retire from the grade system in 1980 at the age of 30.

This age-grade system, as we have described it so far, poses no essential problems for the Galla society. What makes the system interesting to study is that the roles of a male in the tribe depend entirely on which of the five grades he is occupying and not on his age, wisdom, or strength. Two members of the Luba grade, for example, have the same rights and responsibilities, even if one is 7 years old and the other 47.

What are the particular roles assigned to the males in each grade? The anthropologist George Peter Murdock [1959] gives a concise description:

> During the first grade . . . males are forbidden to have sex relations and they wander about begging food, which is always termed "milk" from married women. This is strongly suggestive of the behavior of infants. During the second grade they become initiated into sexual life but without forming stable relationships, and they engage in masked processions and behave generally in an irresponsible manner suggestive of adolescence. In the third grade they serve as warriors and are permitted to marry. Military valor is encouraged in some tribes . . . by requiring the taking of the genitals of a slain enemy as a trophy to qualify for full participation in the activities of the next, or ruling, grade. When an age-set enters the fourth, or Luba, grade, its members take over all important administrative, judicial, and priestly offices in the tribe and run its affairs for eight years . . . . The chief of the age-set, elected when it occupied the second grade, now becomes the high chief of the tribe. Another man becomes speaker of the general assembly. Others assume various administrative and judicial offices—chief priest, finance minister, and so on. During the last, or Yuba, grade, these men relinquish their posts and become "guardians," serving the new officials in a purely advisory capacity.

Murdock's description indicates that the system may have been based, in its origin, on the maturity levels and abilities that corresponded with chronological age; that is, when the age-grade system began, the lower grade was made up entirely of children, while the highest grade was composed of the tribe's elders. As we have seen from our examples, however, in succeeding generations, the relationship between a man's age and the grade he occupies may be very complex.

Since the rules of the age-grade system permit a young man to occupy a high grade, while an older man may be restricted to the activities allowed to members of a lower grade, tensions can easily arise in the tribe. Another anthropologist, Hans Hoffmann [1965], studied this system with the use of mathematical models. He raised the fundamental problem:

> It is evident that the stability of Galla communities is threatened by the arbitrary interval of 40 years that is interposed between generations. Since this interval is often greater than the actual chronological difference between generations, the ages of some of the people in the grades may become progressively greater. This can result in humiliation and incongruity. An old man, entering the first grade, would be required to abstain from sexual activity and to wander around with its youthful members begging food. Further, if he should die before attaining the higher grades, important governmental offices may go unfilled.

The fact that it is possible for a man to "enter" the age-grade system before his actual birth leads to a similar kind of difficulty. He may reach the middle grades of the system at too early a chronological age. He may not be equipped to fulfill the military or ruling functions with any competence. By the time he has the physical strength, talents, and experience to occupy these roles with distinction, he has graduated to the highest grade, where his services are no longer available to the tribe.

Thus, the society may be seriously weakened because it does not have access to the skills of its members at the time it needs them.

"It is curious fact," writes A. H. J. Prins [1953], "peculiar to the Galla institution, that the physical age of those who occupy simultaneously one and the same grade varies so widely, even from young children to fairly old men. . . . Viewed from the institutional angle, what it comes to is that most members of any grade fail to accomplish what is socially expected of them. The grade is supposed to exist because of the expected execution of a delegated task which regards more or less the real ages of the participants, but owing to factual circumstances widely differing from those reflected in the implicit charter, the grades, especially the lower ones, seem to have become an institutional (or even 'functional') failure. This failure has to be attributed to the composition of the personnel."

If there is too high a proportion of males in the society who are "out of phase" with the roles of the age-grade system, it will be difficult to maintain both a strong community and the age-grade system.

Since every community must place a high premium on its own survival, we may well ask if the age-grade system can continue unchanged over a period of many generations. Does the system possess stability as a component of the culture? Must it change to relieve the tensions we have described? Or will the differences between the 40-year intervals and the gaps between successive generations somehow "smooth out" over the years so that these tensions are essentially absent?

Hoffmann developed two models to study these questions. The first [1965] is a relatively simple deterministic model, while the second [1971] is a more sophisticated probabilistic one that makes use of Markov chains.

III. A DETERMINISTIC MODEL

To formulate a mathematical model, we must make some careful definitions and assumptions about the phenomena we hope to study. To investigate whether the age-grade system possesses stability, Hoffmann [1965] first had to make precise the idea of a stable system. He proposed the following:

DEFINITION **A *stable* system is one which tends to maintain a realistic relationship between age and role behavior.**

For his deterministic model, Hoffmann investigated an axiom about sufficient conditions for a system to be stable.

AXIOM 1 **A realistic relationship between age and role behavior can be maintained if, between any arbitrary number of generations, the ages at which an ancestor and his distant offspring entered the first grade are equal.**

This axiom provides the means for translating our verbal discussions about stability into mathematics. Note that the condition for stability is an equality between numbers. We can make this more transparent by introducing some notation.

For the ith generation, we let A_i denote the age at which a man enters the first grade. Thus, A_1 gives the age of the first man of interest when he enters the lowest grade, A_2 the age of his son, A_3 the age of his grandson, and so on. For simplicity, we will assume that each man has exactly one son.

By P_i, we will denote the age of the man in the ith generation when his son is born.

In terms of this notation, we have two ways of writing the age of the man in the first generation at his retirement from the age-grade system. On the one hand, since he enters the system at age A_1 and remains in it for 40 years, he retires at age $A_1 + 40$. On the other hand, since his son enters at the time the father retires, the father's age at retirement is also given by $P_1 + A_2$. Thus, we have the basic relationship,

$$A_1 + 40 = P_1 + A_2, \tag{1}$$

which we may rewrite as

$$A_2 = A_1 + 40 - P_1. \tag{2}$$

If we require that a man and his son enter the age-grade system at the same age, then we are insisting that $A_1 = A_2$. Substituting this equality into Eq. (2) yields

$$40 - P_1 = 0$$

or

$$P_1 = 40.$$

The basic relationship stated in Eq. (2) holds for every pair of father and son; thus we have,

$$A_{i+1} = A_i + 40 - P_i. \tag{3}$$

In particular, this gives us

$$\begin{aligned} A_3 &= A_2 + 40 - P_2 \\ &= (A_1 + 40 - P_1) + 40 - P_2 \\ &= A_1 + 2(40) - (P_1 + P_2) \end{aligned} \tag{4}$$

and

$$\begin{aligned} A_4 &= A_3 + 40 - P_3 \\ &= A_1 + 2(40) - (P_1 + P_2) + 40 - P_3 \\ &= A_1 + 3(40) - (P_1 + P_2 + P_3). \end{aligned} \tag{5}$$

A simple induction argument shows that

$$A_{n+1} = A_1 + n(40) - (P_1 + P_2 + \cdots + P_n). \tag{6}$$

We have seen that a man and his son will enter the age-grade system at the same age exactly if the son is born when his father is 40 years old. From Eq. (4), we may conclude that a man and his grandson will enter the age-grade system at the same age, that is, $A_3 = A_1$, exactly if

$$P_1 + P_2 = 80, \tag{7}$$

or

$$\frac{P_1 + P_2}{2} = 40. \tag{8}$$

This last equation asserts that the average age of parenthood of the first two generations must be 40 if the man and his grandson are to enter the system at the same age.

Now, Axiom 1 asserts that stability is maintained if the ages of entry of a man and his distant descendant are the same. If n denotes a large, arbitrary number of generations, then this condition is expressed by the equality

$$A_{n+1} = A_1. \tag{9}$$

Substituting this equality into Eq. (6) gives

$$A_1 = A_1 + 40n - (P_1 + P_2 + \cdots + P_n), \tag{10}$$

or

$$40n = P_1 + P_2 + \cdots + P_n, \tag{11}$$

so that

$$40 = \frac{P_1 + P_2 + \cdots + P_n}{n}. \tag{12}$$

Now the number on the right-hand side of Eq. (12) is simply the average of the numbers $P_1, P_2, \ldots, P_n$. We may then conclude from this deterministic model that the age-grade system of the Gallas will be stable if, over a large number of generations, the average age at which a man becomes a father is 40.

This deterministic model has the advantage that the predicted condition for stability—average age of 40 for parenthood—can be readily checked by examining accurate census data for the tribe.

This model has a number of important limitations, however. It deals only with one-dimensional father-son links and ignores the branching of descent lines representing siblings. In other words, the model assumes that a man has only one son when, in fact, many men have several sons. Of course, some men have no sons, and this shows another weakness of the model. The model transforms every given family into points of future time when it may, in fact, no longer exist.

Other aspects of this model and possible refinements and improvements of it will be presented in the exercises. In the next section, we will examine Hoffmann's probabilistic model for the question of cultural stability of the age-grade system.

IV. A PROBABILISTIC MODEL

The stability of the Galla age-grade system is threatened by the possibilities of disparities between chronological ages and assigned cultural roles. To promote stability, it is desirable that the lower grades consist largely of adolescents. What is crucial is not the absolute number of members of different ages in a particular grade, but the relative numbers. If most candidates for initiation into the lower grades are youthful, then there will be little tension in the system and we may expect it to continue to function largely unchanged for a number of generations. We can predict the level of tension that is likely to arise in the future if we know the ages of the males at the time they enter the age-grade system.

For computational simplicity, we will consider three age categories, or states, for the age at the time of initiation into the lowest grade:

S_1: ages 13–19,

S_2: ages 20–29,

S_3: ages 30 or over.

The vector (x_1, x_2, x_3) will represent the proportion of males in each state. For example, if there are 100 men about to be initiated into the age-grade system with 25 in S_1, 55 in S_2, and 20 in S_3, we will represent this by the vector

$$(25/100, \quad 55/100, \quad 20/100) = (.25, \quad .55, \quad .20). \tag{13}$$

If the Galla system is to survive, the set of males about to enter the grade system should consist largely of younger men. The state S_1 should contain a relatively large proportion of the set while S_3 should include a relatively smaller fraction. As new generations enter the system, there should not be a significant drift from S_1 to S_3.

To determine the shifts from one state S_i to another S_j in successive generations, we determine for each male about to enter the system, the age of his father when the father entered the system.

Using Hoffmann's example, suppose that we examine a set of 240 males and record for each his state and his father's state at the time of initiation. The data are conveniently displayed in a matrix in which the rows correspond to the father's state and the columns to the son's state:

$$\begin{array}{rc} & \text{Son's state at time of initiation} \\ & \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{array}{r} \text{Father's} \\ \text{state at time} \\ \text{of initiation} \end{array} \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 10 & 25 & 30 \\ 55 & 60 & 35 \\ 5 & 15 & 5 \end{pmatrix} \end{array}$$

We see from this matrix that the largest group (60) were in their 20's when they were initiated and so were their fathers. There were only 5 males who were initiated after the age of 30, but whose sons were initiated in their teens.

As noted above, our concern is not so much with absolute numbers, but with proportions. If we examine the 65 fathers who were initiated into the age-grade system as teenagers, we see that $\frac{10}{65}$ of them had sons who were initiated as teenagers, $\frac{25}{65}$ had sons initiated in their twenties, and $\frac{30}{65}$ had sons initiated after the age of 30. We compute similar fractions for the fathers in states S_2 and S_3 and obtain the matrix

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{pmatrix} 10/65 & 25/65 & 30/65 \\ 55/150 & 60/150 & 35/150 \\ 5/25 & 15/25 & 5/25 \end{pmatrix} \end{array} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{pmatrix} 2/13 & 5/13 & 6/13 \\ 11/30 & 12/30 & 7/30 \\ 1/5 & 3/5 & 1/5 \end{pmatrix} \end{array}$$

or, in decimal notation,

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{pmatrix} .154 & .384 & .462 \\ .367 & .4 & .233 \\ .2 & .6 & .2 \end{pmatrix} \end{array}$$

Now it is possible to regard the entries in the matrix as probabilities. Thus the probability that a father in state S_2 has a son in state S_3 is given as .233. Hoffmann considers this matrix as a transition matrix from one generation to the next and notices that if we assume that this matrix remains constant, then we can study the Galla age-grade system using the tools of Markov chain analysis.

For example, if our initial distribution of states is given by the vector

$$\mathbf{p}^{(0)} = (.25, .55, .20),$$

then the distribution of states after one generation is

$$\mathbf{p}^{(1)}P = (.28, .44, .28).$$

After a single generation, there will be a slightly higher proportion of sons in S_3 than there were sons a generation ago. If this trend continues, the stability of the age-grade system is threatened.

Let's see what happens to the distribution after two generations. It will be given by the vector $\mathbf{p}^{(2)}$ where

$$\mathbf{p}^{(2)} = \mathbf{p}^{(1)}P = (\mathbf{p}^{(0)}P)P = \mathbf{p}^{(0)}P^2 = (.27, .46, .27).$$

It is easy to see that the drift from S_1 to S_3 evidenced after one generation has not continued.

In a similar fashion, we can compute the distribution of states after 3 generations, 4 generations, and so on. It is more interesting at this point, however, to determine the long range behavior of the distribution vector. All of the entries of the transition matrix P are positive, so we are dealing with a regular Markov process. The long-term distribution of states is then given by the unique fixed-point stochastic vector $\mathbf{w}$ of P. This is the vector $\mathbf{w} = (w_1, w_2, w_3)$ with the properties that $\mathbf{w} = \mathbf{w}P$ and $w_1 + w_2 + w_3 = 1$. Using the methods of Chapter 10 and Appendix II, we find that the components of $\mathbf{w}$ are

$$w_1 = \frac{663}{2518} = .263,$$

$$w_2 = \frac{1140}{2518} = .453,$$

$$w_3 = \frac{715}{2518} = .284.$$

If our process is a Markov chain, then the proportion of males in the three states will tend toward $S_1 = .263$, $S_2 = .453$, $S_3 = .284$. Hoffmann notes that these values are not radically different from the initial vector (.25, .55, .20) so that the system may be considered stable.

V. CRITICISMS OF THE MODELS

In what sense is Hoffmann correct in claiming that the mathematical model predicts that the Galla age-grade system is stable? In the first place, the long-term behavior of the distribution is close to the distribution of the initial vector. If the society was able to tolerate the distribution of states when the system began, it will be able to tolerate distributions just as well in later generations. Even if the initial distribution vector was quite different from $\mathbf{w}$, there is still reason to conclude the system is stable. The age-grade system is most threatened if the proportion of older men in the lower grades continues to increase. The calculation of the limiting vector $\mathbf{w}$ shows that this proportion will remain, in the long run, under 30 percent. Whether or not the society can tolerate that high a proportion in the lowest age group is a question that can be decided only by more careful observation of the Gallas.

The discussion of the particular numbers obtained in Hoffmann's example is not central to a criticism of his approach. It should be pointed out that the data he used were not obtained by an actual observation or census of the Gallas, but were chosen arbitrarily. Hoffmann wished to demonstrate how Markov chains could be used to study a problem of cultural stability. The entries of the transition matrix were chosen to represent a not unreasonable situation for which no bias toward or away from stability was immediately apparent.

The critical assumption in Hoffmann's model is that the process is a Markov one; that is, that the transition matrix remains constant from generation to generation over many years. How reasonable is this assumption? Are the transition probabilities going to be the same for a generation of fathers that suffered through a drought or were decimated by illness or war as they are for a generation that has known plentiful harvest, good health, and peace? If one generation produces a set of males most of whom enter the lowest grade at an advanced age, will the next generation try to adjust its birth rates to compensate for this condition?

Hoffmann's response to these criticisms is that there is value in the Markov chain approach even if there is no reason to believe that the transition matrix is constant [1971]: "If we are unwilling to postulate the invariance of the transition matrix, it is still possible to use the model as a decision procedure. The limiting vector of an observed transition matrix is readily calculated. Then one can state: 'This pattern of transitions is/is not compatible with the stability of the . . . system.'"

If we are willing, on the other hand, to postulate that the transition matrix remains constant, we can ask some important questions about Hoffmann's Markov chain model. In our early discussions about the age-grade system, we noted that it is quite conceivable that many males will enter the lowest grade before adolescence. This group is omitted entirely from Hoffmann's model. For completeness, he could have included a state S_0 corresponding to those who were initiated before the age of 13. If this state is included, then the transition matrix becomes a 4×4 array. If it is a regular matrix, then the theory of Chapter 10 still holds and a limiting vector $\mathbf{w}$ can be computed, although the calculations are more tedious.

In the exercises and suggested projects, you will be asked to explore further some possible modifications of this probabilistic model.

VI. HANS HOFFMANN

Hans Hoffmann's work in anthropology has ranged from field studies of Eskimo hunters and the cultures of the Amazon River basin to new theoretical developments in mathematical anthropology. He has also conducted ethnographic research among a mental hospital population receiving new psychiatric drugs, and he has served as a consultant on a project attempting a mathematical analysis of children's games.

Hoffmann was born in Koblenz, Germany in 1929, but received his professional education in the United States. He did his undergraduate work at Cornell University where he was a mathematics major who devoted substantial

Hans Hoffmann and two of his children.

time also to the fields of physics, astronomy, anthropology, and Chinese literature. He received his doctorate in anthropology from Yale University in 1957 for a thesis on cultural homogeneity among the Attawapsikat Cree. The field data for this study, which was supervised by an anthropologist and a psychologist, were gathered by Hoffmann and his wife Betty in James Bay, Canada.

Interest in the hunters of the northern forest led Hoffmann to the field work among the Eskimos and Crees in the mid-1950's. Among the unpublished material Hoffmann collected are reminiscences and ethnographic comments by his Cree informant gathered while Hoffmann observed his life in camp and accompanied him on hunting expeditions.

The Amazon River basin has also held a long-term fascination for Hoffmann. "So far," he writes, "this has led to three field trips to the Shipibo of the Ucayali river in Eastern Peru. I am particularly interested in contemporary changes in technology and their effects on the continuity of Amazonian cultures. This research is illuminated by an independent interest in lowland archeology. Several of my studies in mathematical anthropology are based on my Amazonian data. Conversely, mathematical culture theory has supplied an analytical framework for making field observations."

Much of Hoffmann's recent work has been devoted to mathematical models in theoretical anthropology. He has published papers on deterministic, stochastic, and game theory models of cultural systems, material culture in a four dimensional world, linear programming approaches to "cultural intensity," and an extensive survey of mathematical systems and their possible application to anthropological problems.

In a chapter for the *Biennial Review of Anthropology*, Hoffmann [1969] describes the role of mathematical models:

> Although human imagination is unbounded, our unaided ability to experience it is limited. Experiencing requires tools, and as these become developed, wider realms of imagination can be made one's own. We can imagine differences in the length of objects, but need a tool—the natural numbers—to experience them. We can imagine an infinity of numbers beyond the integers and their inverses, but need a tool—Cantor's diagonal proof—to experience their existence. For this reason, tools are the essence of culture; whether physical or mental, they permit man to experience wider ranges of his universe . . . Mathematics is a tool that enables man to understand and control an immense number of events and processes in the physical world. Mathematics, in particular, is a tool that penetrates realms of imagination hopelessly beyond the experience of a toolless mind. Moreover, once mathematical tools have been developed, they often reverse their effect and enlarge not only one's experience but also one's imagination.

Hoffmann has taught at the University of Oklahoma, University of Arkansas, Cornell, and the State University of New York at Binghamton, where he is now Professor of Anthropology. "From a long-range point of view," he says, "I do expect to return to empirical investigations when the mathematical issues that have intrigued me have been at least looked into. While mathematics is an exhilarating world to explore, it cannot really compete with the Amazon. Most likely I will combine a long standing interest in the construction and sailing of small boats with that in the life of Indian traders on the Amazon tributaries. . . . I look forward to returning to the Amazon when the ages of my children make extensive field work feasible once more."

EXERCISES

II. THE GALLA SYSTEM

1. Meyer entered the age-grade system at the age of 15 and his son Frank was born when Meyer was 35 years old. Frank became the father of Michael at age 39. Michael's sons Eli and Alexander were born when he was aged 25 and 31, respectively. How old will Eli and Alexander be when they retire from the system?

2. Is it possible for a man to be born directly into the highest grade? Is it possible for him to be born after his "retirement" from the age-grade system?

III. A DETERMINISTIC MODEL

3. Prove Eq. (6) by mathematical induction.

4. What is the relation of A_{n+1} and A_1 if the average age of parenthood of the intervening n generations is less than 40 years? greater than 40 years?

5. How do the results of the deterministic model change if the length of time of an individual in the age-grade system is k years, instead of 40 years?

6. What would you estimate the life expectancy of an Ethiopian tribesman to have been in the eighteenth and nineteenth centuries? Is it likely that the average of parenthood could have been 40?

7. Note that the rules of the age-grades do not allow a man to marry until he has been in the system for at least 16 years. What effect does this have on the average age of parenthood?

8. How would you modify the deterministic model to allow for the fact that some men have no sons, while others have more than one?

9. If the society is undergoing exponential or logistic population growth, will this affect the stability of the age-grade system?

10. Prins argued that "the functioning of the system of age-grades of the Galla . . . requires birth regulation as one of its basic institutional elements."

a) Show that the results of Hoffmann's deterministic model indicate that this is *not* a necessary condition for stability.

b) Prins claimed that restricting procreation to a man's last 12 years in the system would be the ideal way to achieve stability. In what sense, if any, is this true?

IV. A PROBABILISTIC MODEL

11. Using Hoffmann's transition matrix P, calculate $\mathbf{p}^{(1)}, \mathbf{p}^{(2)}$, and $\mathbf{p}^{(3)}$ if

a) $\mathbf{p}^{(0)} = (1,0,0)$

b) $\mathbf{p}^{(0)} = (\frac{1}{3},\frac{1}{3},\frac{1}{3})$

c) $\mathbf{p}^{(0)} = (0,\frac{3}{4},\frac{1}{4})$

d) $\mathbf{p}^{(0)} = (p, q, 1-p-q)$

12. Repeat Exercise 11 with the transition matrix

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{pmatrix} .6 & .3 & .1 \\ .1 & .8 & .1 \\ .1 & .2 & .7 \end{pmatrix} \end{array}$$

13. Repeat Exercise 11 if every entry in the transition matrix is $\frac{1}{3}$.

14. Find the unique fixed point stochastic vector for

a) the matrix M of Exercise 12,

b) the matrix of Exercise 13.

Are these stable systems?

15. Add a fourth state S_0 for those who entered the system before age 13, assume that $\mathbf{p}^{(0)} = (.1, .2, .3, .4)$ and that the transition matrix is

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{pmatrix} .5 & .3 & .15 & .05 \\ .2 & .6 & .15 & .05 \\ 0 & .2 & .7 & .1 \\ 0 & .05 & .15 & .8 \end{pmatrix} \end{array}$$

a) Compute $\mathbf{p}^{(1)}$ and $\mathbf{p}^{(2)}$

b) Is the transition matrix regular? If so, find its unique fixed-point stochastic vector. Does it give rise to a stable system?

16. Is it conceivable that the transition matrix might not be regular? What are the consequences of this for the model?

SUGGESTED PROJECTS

1. What are the effects on the stability of the age-grade system if one or more of the following modifications are made?

a) A son enters the system at his father's death if his father dies before retirement;

b) The eldest son enters exactly 40 years after his father does, but younger sons must wait until they are the same age as their older brother was when he was initiated;

c) A son enters 40 years after his father does or at the age of 10, whichever event takes place later; thus, no one enters before the age of 10.

Can the deterministic and probabilistic models of this chapter be changed to incorporate these variations? Are new models necessary?

2. Discuss the practical problems of determining the entries of the transition matrix from observations and census data. Can you determine, in the absence of such information, bounds for the sizes of the entries of the transition matrix? Are all transition matrices whose entries satisfy these bounds necessarily regular?

3. Prins discusses age-grade systems among the Kipsigis and Kikuyus of Kenya as well as the Gallas of Ethiopia. Develop mathematical models for these age-grade systems. Do these systems have problems of stability?

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12 Paired-Associate Learning

The mind is slow to unlearn what it has been long in learning.

Seneca