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MAT 375

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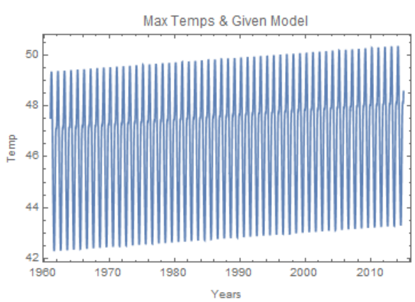
Lome Final Model Fit to City

Donna Odhiambo and I analyzed rainfall and temperature data for Lome, Togo. For the third mini project we came up with a model to predict rainfall over time in Lome. We later used the modeling team's final model and applied it to our data to see how it fit and come away with conclusions. Our first rainfall model was not extremely accurate, but we improved it upon using the new model developed by the modeling team which included new factors such as the El Nino Southern Oscilations, longitude, latitude, Sea surface temperature, and elevation.

Maximum Temperatures

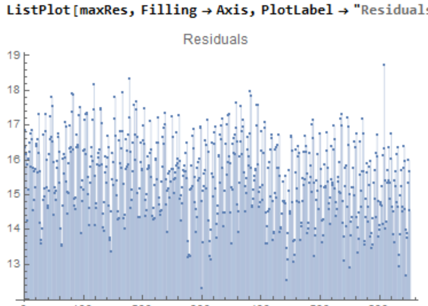
```
In[13]:= maxData = Transpose[{years, maxTemp}];
maxPlot = ListPlot[maxData, PlotStyle → Red];
Show[Plot[maxModel[decYear], {decYear, 1961, 2015}, PlotLabel → "Max Temps & Given Model", Frame → True, FrameLabel → {"Years", "Temp"}], maxPlot]
```

Out[15]=




```
In[16]:= calcMax = maxModel[#] & /@ years;
maxRes = calcMax - maxTemp;
ListPlot[maxRes, Filling → Axis, PlotLabel → "Residuals"]
```

Out[18]=



```
In[19]:= Min[calcMax]
Out[19]= 42.4266

In[20]:= Max[maxTemp]
Out[20]= 34.9
```

The given model for maximum temperatures completely overshoots all of the maximum data points for Lome. The model for maximum temperatures gives the minimum temperature in the range 1961-2016 to be 42.4456 Degrees Celcius, while the maximum temperature in the actual dataset is 38 Degrees Celcius.

Here are the result from our first attempt:

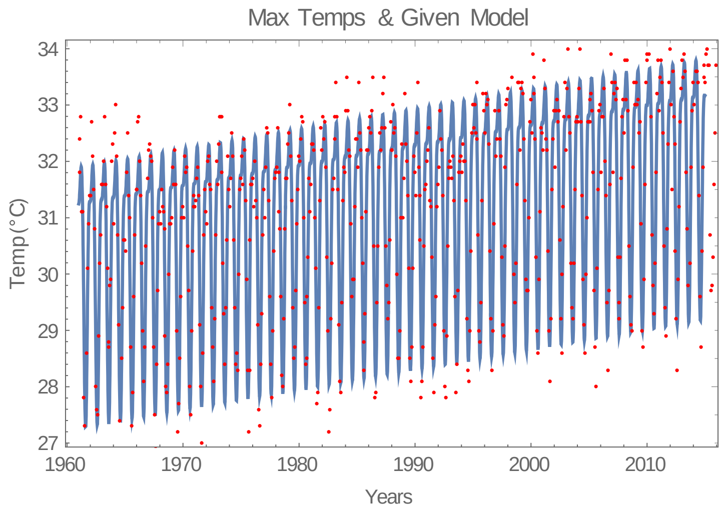
```
m1 = NonlinearModelFit[maxData, a Sin[2 π x] + b Cos[2 π x] + c Sin[4 π x] + d Cos[4 π x] + e Sin[6 π x] + f Cos[6 π x] + g x + h lat + i (2 lat lon) + j lat^2 + k lon + l lon^2 + m elev, {a, b, c, d, e, f, g, h, i, j, k, l, m}, x]
m1["BestFitParameters"] // TableForm
m1["ParameterTable"] // Quiet
m1["ParameterConfidenceIntervals"] // MatrixForm // Quiet

FittedModel[ -39.6357 +0.0355833 x +1.57328 Cos[2 π x] -0.379756 Cos[4 π x] -0.0904069 Cos[6 π x] +1.34853 Sin[2 π x] -0.687535 Sin[4 π x] +0.0128698 Sin[6 π x] ]

bleForm=
a → 1.34853
b → 1.57328
c → -0.687535
d → -0.379756
e → 0.0128698
f → -0.0904069
g → 0.0355833
h → -0.674077
i → -0.311113
j → -0.0687834
k → -6.09781
l → -5.62875
m → -0.0142986
```

	Estimate	Standard Error	t-Statistic	P-Value
a	1.34853	0.0302557	44.571	2.98425×10^{-198}
b	1.57328	0.0302852	51.9488	1.45195×10^{-231}
c	−0.687535	0.0304201	−22.6013	1.44719×10^{-83}
d	−0.379756	0.0301137	−12.6107	1.02059×10^{-32}
e	0.0128698	0.0302703	0.425162	0.670862
f	−0.0904069	0.0302665	−2.98703	0.00292492
g	0.0355833	0.00135575	26.2463	1.41427×10^{-103}
h	−0.674077	0.000118366	−5694.88	$1.257058199064 \times 10^{-1505}$
i	−0.311113	0.000256459	−1213.11	$1.527839702537 \times 10^{-1076}$
j	−0.0687834	0.00115998	−59.2969	5.68948×10^{-262}
k	−6.09781	0.0000130846	−466029.	$5.507581115657 \times 10^{-2728}$
l	−5.62875	0.000014175	−397090.	$1.468120725395 \times 10^{-2683}$
m	−0.0142986	0.00558009	−2.56243	0.0106222

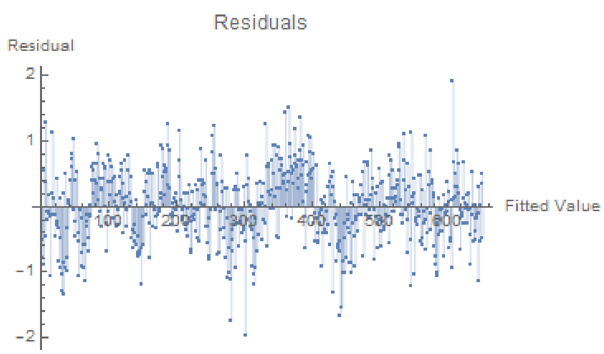
Below, we can see that the trend of the model generally fits the data. The red data points for temperature appear to be top heavy indicating that our temperatures are most likely rising as the model suggests:



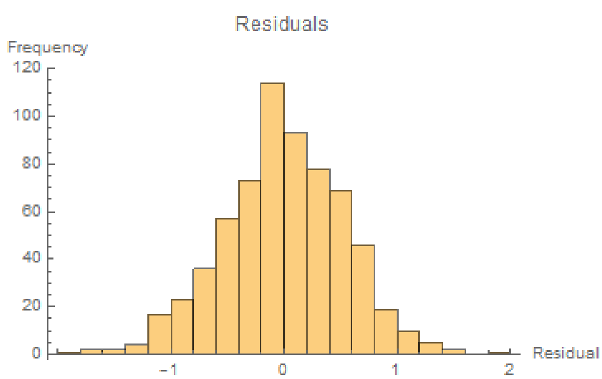
Here are the residuals which look to be normally distributed:

```
In[26]:= calcMax = m1[#] & /@ years;
maxRes = calcMax - maxTemp;
ListPlot[maxRes, Filling → Axis, PlotLabel → "Residuals", AxesLabel → {"Fitted Value", "Residual"}]
Histogram[maxRes, PlotLabel → "Residuals", AxesLabel → {"Residual", "Frequency"}]
```

Out[28]=



Out[29]=

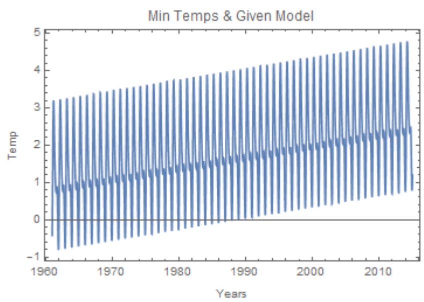


Next we observed minimum temperatures. Using the same model form with different coefficients yielded a much better fit, however the parameter tied directly with decimal Year, g, and with elevation, m, turned out to be insignificant. The residuals however seem random and have near normal distribution about zero, more so than the residual outcomes for the maximum temperatures. Incorporation of sine & cosine pairs with arguments of 8\[Pi]x and larger turned out to be insignificant. The general trends are well captured by arguments of 2\[Pi]x, 4\[Pi]x, and 6\[Pi]x. Model is demonstrated below:

Minimum Temperatures

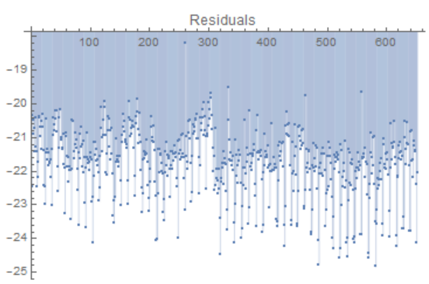
```
In[39]:= minData = Transpose[{years, minTemp}];
minPlot = ListPlot[minData, PlotStyle → Red];
Show[Plot[minModel[decYear], {decYear, 1961, 2015}, PlotLabel → "Min Temps & Given Model", Frame → True, FrameLabel → {"Years", "Temp"}], minPlot]
```

Out[41]=




```
In[42]:= calcMin = minModel[#] & /@ years;
minRes = calcMin - minTemp;
ListPlot[minRes, Filling → Axis, PlotLabel → "Residuals"]
```

Out[44]=




```
Max[calcMin]
4.81626

Min[minTemp]
19.2
```

```
m3 = NonlinearModelFit[minData, a Sin[2 π x] + b Cos[2 π x] + c Sin[4 π x] + d Cos[4 π x] + e Sin[6 π x] + f Cos[6 π x] + g x + h lat + i (2 lat lon) + j lat^2 + k lon + l lon^2 + m elev, {a, b, c, d, e, f, g, h, i, j, k, l, m}, x]
m3["BestFitParameters"] // TableForm
m3["ParameterTable"] // Quiet
m3["ParameterConfidenceIntervals"] // MatrixForm // Quiet
```

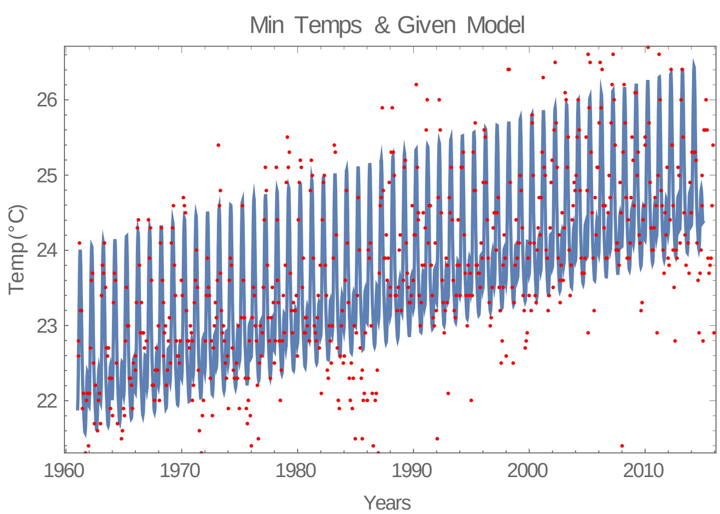
FittedModel[−67.7216 + 0.0459749 x + 0.181852 Cos[2 π x] − 0.513443 Cos[4 π x] − 0.210527 Cos[6 π x] + 0.861406 Sin[2 π x] − 0.0907043 Sin[4 π x] − 0.144408 Sin[6 π x]]

bleForm=

```
a → 0.861406
b → 0.181852
c → -0.0907043
d → -0.513443
e → -0.144408
f → -0.210527
g → 0.0459749
h → -1.15173
i → -0.531567
j → -0.117523
k → -10.4187
l → -9.61727
m → -0.0244306
```

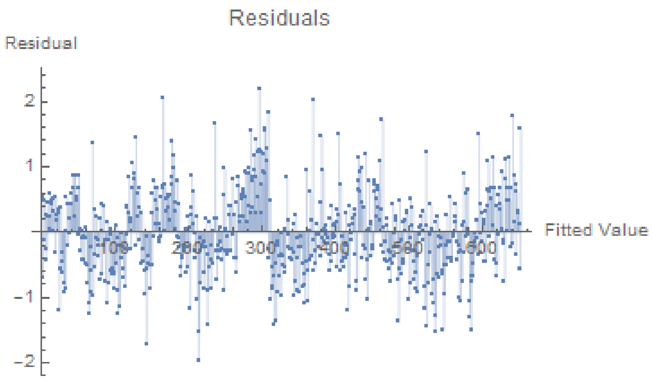
	Estimate	Standard Error	t-Statistic	P-Value
a	0.861406	0.0373726	23.0491	5.1369×10^{-86}
b	0.181852	0.0374091	4.86118	1.47024×10^{-6}
c	−0.0907043	0.0375757	−2.41391	0.0160625
d	−0.513443	0.0371973	−13.8032	4.04539×10^{-38}
e	−0.144408	0.0373906	−3.86215	0.000123841
f	−0.210527	0.0373859	−5.63119	2.68261×10^{-8}
g	0.0459749	0.00167465	27.4534	3.37398×10^{-110}
h	−1.15173	0.000146208	−7877.32	$1.173230461490 \times 10^{-1595}$
i	−0.531567	0.000316784	−1678.01	$1.518827832427 \times 10^{-1166}$
j	−0.117523	0.00143284	−82.0212	$1.934831463667 \times 10^{-341}$
k	−10.4187	0.0000161625	−644624.	$5.124907830850 \times 10^{-2818}$
l	−9.61727	0.0000175093	−549266.	$1.366114182672 \times 10^{-2773}$
m	−0.0244306	0.00689267	−3.54443	0.000422181

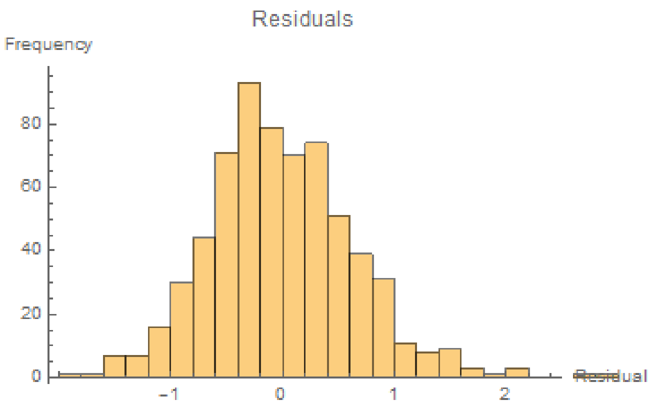
Looking at the graph below, we get a glimpse of how well the model is predicting. Just by quickly observing, we see that there are a a low percentage of data points outside of our model range so it appears that the model would be suitable.



Residuals:

```
calcMin = m3[#] & /@ years;
minRes = calcMin - minTemp;
ListPlot[minRes, Filling → Axis, PlotLabel → "Residuals", AxesLabel → {"Fitted Value", "Residual"}]
Histogram[minRes, PlotLabel → "Residuals", AxesLabel → {"Residual", "Frequency"}]
```





The parameter tied directly with decimal Year, g, and with elevation, m, turned out to be insignificant. The residuals however seem random and have near normal distribution about zero, more so than the residual outcomes for the maximum temperatures. Incorporation of sine & cosine pairs with arguments of 8\[Pi]x and larger turned out to be insignificant. The general trends are well captured by arguments of 2\[Pi]x, 4\[Pi]x, and 6\[Pi]x.

Summary

The modeling group's model, in general, fit our city well. Overall, it did a better job of capturing the trends of minimum and maximum temperatures than we could come up with in our previous model. I do think if we had more time to look at other potential factors such as wind patterns, local pollution levels and other environmental hazards, we would be able to come up with an even more accurate model. The final model had a wider range of variability and was able to capture and explain many of the outliers we initially had. Also, most aspects of nature are nonlinear, so adding the complexity of additional variables (ENSO, SST, longitude, latitude) opened up more potential for our predictions to be precise and accurate.