Togo, West Africa: on Track for Over Three Degrees Celsius of Warming by 2100

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Abstract. Togolese meteorologists (Direction de la Météorologie Nationale, Division Climatologique) have kept temperature records on ten cities and towns since 1961. These records (monthly average maxima and minima) permit an analysis of temperature trends through time. We also have monthly rainfall records, and so consider changes in rainfall over time.

Our results indicate that Togo has been warming since 1961 at a rate of around two and a half degrees Celsius per 100 years, which suggests that Togo is on track for three-plus degrees of warming over pre-industrial levels by the end of this century.

The model confirms a trend of cooling by elevation of about 5 degrees per kilometer. Furthermore, the effect of moving north from the ocean is mixed: while there is a slight **cooling** trend by increasing latitude in the minima, there is a much stronger **warming** trend by increasing latitude in the maxima, suggesting a change in rainfall variation. Interestingly enough, there is also a mixed longitudinal impact, with increasing minima from east to west, but decreasing maxima from east to west.

Togo is interested in the implications of climate changes generally, temperature and rainfall in particular, and the consequences for public health and diseases such as malaria, meningitis, cholera, and dengue fever. We are able to make some general statements based on the results of our analysis.

1. Introduction

1.1. Background and Context

Togo gained its independence from France in 1960, and began carrying out the ordinary functions of a sovereign state such as the collection of important meteorological data on its cities and towns. We use the temperature and rainfall records of ten of those towns and cities in this analysis: Lome, Tabligbo, Kouma-Konda, Atakpame, Sotouboua, Sokode, Kara, Niamtougou, Mango, and Dapaong. They provide excellent coverage of Togo, from its southern-most city of Lome, to its northern-most city of Dapaong.

We have records on temperature extremes (average maximum and minimum temperature) by month, and eight of the records cover the period 1961 to 2015 (Kara is available only from 1977, and Sotouboua only from 1982). We combine all the data to construct a single model for each of the maximum and minimum temperature for the country, which we have used to create temperature maps as well as to forecast Togo's future temperature.

Rising temperature has direct implications for health, as well as potential indirect implications: for example, if increasing temperatures impact rainfall, that has obvious consequences for agriculture, say, and less obvious consequences for public health (e.g. more flooding, or more or less standing water). We study the impact of temperature on rainfall data, which we have monthly for roughly the same periods.

1.2. Prior Relevant Temperature Modeling

2. Materials and Methods

Initial data were annual mean maximum and minimum temperatures, and our preliminary inspection led us to conclude that, while it appeared that there is an increase across Togo, we needed to increase the resolution of the data. So we acquired finer measurements from the Togolese Direction de la Météorologie Nationale, and then modeled each location individually. Our results indicated that, although there was wide variation across Togo, the general trends were for increasing temperatures, with a strong oscillatory seasonal component.

We also requested and received elevation, latitude, and longitude for each site: our supposition was that elevation and latitude would play important roles, and that longitude might be important as well. In any event, since these three variables are known for any point in Togo using a simple GPS, a model built including those variables would be easy to use.

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2.1. Data Cleaning

2.2. Temperature Model Development

2.3. Rainfall Model Development

Previous work(*Husak et al.* [2006]; *Yoo et al.* [In Review, 2018]; *Ye et al.* [2018]) suggests that the two-parameter gamma distribution has been used (with mixed success) for modeling rainfall.

One of the problems of the gamma distribution is that zero rainfall values are not permissible within the probability density function (pdf), so a mixed model for rainfall must be adopted (the probability of no rainfall within a month, versus a gamma distribution of non-zero rainfall). Our objective is to model rainfall by month, and determine if there has been a trend toward either more rainfall or change in the variation of rainfall with time (which we take as a surrogate for temperature).

The two-parameter Gamma distribution $Gamma(\alpha, \beta)$ has mean $\alpha * \beta$ and variance $\alpha\beta^2$(*Husak et al.* [2006]). We suggest that the general trend in Togo is towards rainfalls best modeled by $Gamma(\alpha(t), \beta(t))$, where α, β, or both are changing with time.

In the special case where the daily rainfall is a given by $Gamma(\alpha_{day}, \beta_{month})$ (where *day* is the day of the month), then the accumulated daily rainfall given monthly (i.e., our rainfall data) is a sum of gamma distributions, and hence also gamma[1]:

$$Rainfall(year, month) \sim Gamma\left(\sum_{i=1}^{days} \alpha_i, \beta_{month}\right)$$

3. Results

3.1. Temperature Model

Elevation has been shown to have a significant effect on temperature(*Khandelwal et al.* [2017]), on the order of $5 - 10^oC$ per kilometer in that study of Jaipur, India. Our model confirms this (on the lower end), with the temperature dropping approximately 4.95 ± 0.07 degrees/km for maxima, 5.74 ± 0.09 degrees/km for minima.

3.2. Rainfall Model

3.3. City-by-City Validation and Discussion: Sensitivity Analysis

In the end, our model attempts to predict the temperature and rainfall of any city in Togo. It should certainly, then, predict fairly well the results for any individual city whose meteorological data was used in the creation of the model. In this section, we predict the results for each of the cities in our study, and compare the predicted to the original data.

3.3.1. Lomé

3.3.2. Tabligbo

3.3.3. Kouma-Konda

3.3.4. Atakpamé

3.3.5. Sotouboua

3.3.6. Sokodé

3.3.7. Niamtougou

3.3.8. Kara

3.3.9. Mango

3.3.10. Dapaong

4. Discussion

4.1. Health Impacts of Increasing Temperature

4.2. Effect of Increasing Temperature on Rainfall, and Health Impacts

4.3. Summary

Appendix A: Data Treatment and Analysis Details

A1. Problems Encountered

A2. To Remove or Impute (Estimate?)

Appendix B: Outlier Detection

B1. Methods

B2. Outlier Summary

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Notes

1. (we'll need a reference, better than https://math.stackexchange.com/questions/2 of-independent-gamma-distributions-is-a-gamma-distribution !:).

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Table 1. Geographical Coordinate information for the cities and towns of Togo.

There are roughly four and a half degrees of latitudinal difference, and one and a quarter degree of longitudinal difference in these towns. Minimum elevation is about 20m (Lomé), and maximum elevation is around 640m (Kouma-Konda).

Longitude	Latitude	Elevation (m)	Location
1.25315	6.17455	19.60	Lomé
1.50789	6.60475	40	Tabligbo
0.572	6.95617	641	Kouma-Konda
1.12143	7.53838	400	Atakpamé
0.98	8.56	380	Sotouboua
1.15176	8.99517	387	Sokodé
1.20357	9.62788	342	Kara
1.083333	9.8	462	Niamtougou
0.46916	10.36191	146	Mango
0.22809	10.836	230	Dapaong