Section 10.1: Parametric Equations

Curves defined by parametric equations

Example

The location for Tara the tyrannosaurus rex at time t hours is given by

$$(x, y) = (\sin(t) - \sin(2t), \cos(t))$$

Time “t” is the parameter, and once it is known, the dinosaur’s position *(x,y)* is known.

Questions

- What is Tara’s location at time $t = 0$?
- What is Tara’s location at time $t = \pi/2$?
- When does Tara return to the location she was at when t was 0?

Definition

Parametric equations are functional values for x and y coordinates

$x = f(t)$

$y = g(t)$

Questions

Consider the parametric equations $x = \cos(t)$, $y = \sin(t)$.

- Make a table of values for these parametric equations for $t = 0, \pi/6, \pi/4, \pi/3, \pi/2, 2\pi/3, 3\pi/4, 5\pi/6, \pi$. Then connect these points.
- Using the Pythagorean identity, find an equation on x and y these parametric equations satisfy.
- What curve do these parametric equations represent?

Definition

A curve is defined by parametric equations, if all the points on the curve can be represented by parametric equations $x = f(t)$, $y = g(t)$ for different values of t. Graphing calculators and software like *Mathematica* can draw curves represented by given parametric equations.

Example

To draw the curve in *Mathematica* given by the parametric equations

$$x = \sin(t) + \frac{1}{2}\cos(5\,t) + \frac{1}{4}\sin(13\,t)$$

$$y = \cos(t) + \frac{1}{2}\sin(5\,t) + \frac{1}{4}\cos(13\,t)$$

for $0 \le t \le 2\,\pi$ enter

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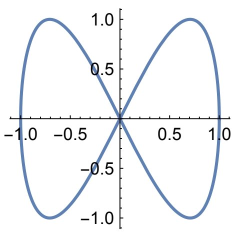
ParametricPlot[
 {Sin[t] + 1/2 Cos[5 t] + 1/4 Sin[13 t], Cos[t] + 1/2 Sin[5 t] + 1/4 Cos[13 t]},
 {t, 0, 2 Pi}
]
```

It’s kind of nice to watch the curve get traced out, which we can do with a “Manipulate” command in Mathematica.

Example: What’s the equation of Infinity?



Identifying curves defined by parametric equations

One may be able to identify a curve defined by parametric equations by finding an equation in only x and y.

Techniques

For the parametric equations $x = f(t)$, $y = g(t)$, two techniques for finding an equation on x and y:

- Find an identity between $f(t)$ and $g(t)$.
- Solve $x = f(t)$ for t and plug this value of t into $y = g(t)$, or vice versa.

Questions

- For parametric equations $x = 2\cos(t)$, $y = 2\sin(t)$, use the Pythagorean identity, $\cos^2(t) + \sin^2(t) = 1$. What do you get?
- For parametric equations $x = 2\,t$, $y = 1 + t^2$, solve $x = 2\,t$ for t. What do you get?

Questions

Identify the curves defined by the parametric equations.

- $x = 2\,t + 1$, $y = 3\,t - 4$

- $x = \cos(t) - 5$, $y = \sin(t) + 6$.
- $x = 2\cos(t)$, $y = 3\sin(t)$.