Section 11.3: Integral Test

Code used below (thanks to Al Hibbard)

Review

Question

What are the first four partial sums for $\sum_{k=1}^{\infty} \frac{1}{k^2}$?

Questions

Which of the following converge? To what?

- $\sum_{k=0}^{\infty} 4\left(\frac{3}{2}\right)^k$
- $\sum_{k=0}^{\infty} 5\left(-\frac{2}{3}\right)^k$
- $\frac{7}{4^3} + \frac{7}{4^4} + \frac{7}{4^5} + \frac{7}{4^6} + \ldots$

Question

How do I know that $\sum_{k=1}^{\infty} \sin(k)$ diverges?

Integral Test

Questions

Consider the series $\sum_{k=1}^{\infty} \frac{1}{k^2}$. The summands of this series are $1, \frac{1}{4}, \frac{1}{9}, \ldots$.

- You have computed the first few partial sums for this series. How do the partial sums compare, S_1 with S_2, S_2 with S_3, etc.?
- Is this monotonic or not?

Questions

Suppose the summands $a_1, a_2, a_3, \ldots$ are all positive.

- Are the partial sums $S_n = \sum_{k=1}^{n} a_k$ monotonic?
- If we can find an upper bound for the sequence $S_1, S_2, S_3, \ldots$ what do we now about the convergence of $\sum_{k=1}^{\infty} a_k$?

Questions

Let $a_k = \frac{1}{k^2}$. The plot of these terms is

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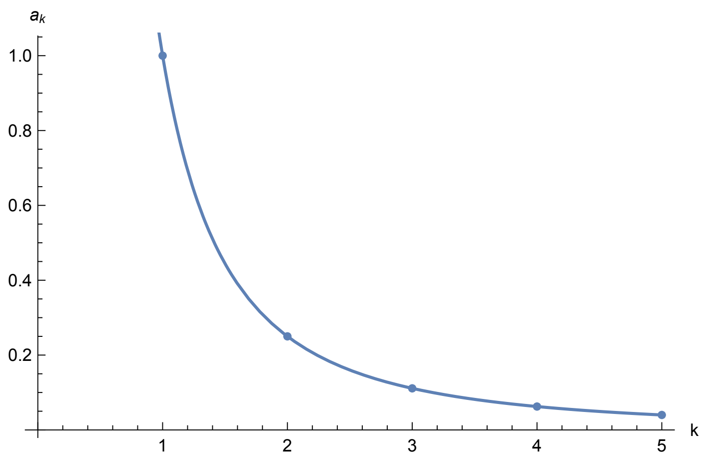

- Approximate $\int_1^5 \frac{1}{x^2}\,dx$ using $n = 4$ and the right hand endpoints.
- Is that an underestimate, overestimate, or can't tell?
- How is that approximation related to $\sum_{k=2}^{5} \frac{1}{k^2}$?
- What is $\int_1^\infty \frac{1}{x^2}\,dx$? What can you conclude about $\sum_{k=1}^{\infty} \frac{1}{k^2}$?

Questions

Let $a_k = \frac{1}{k}$. The plot of these terms is

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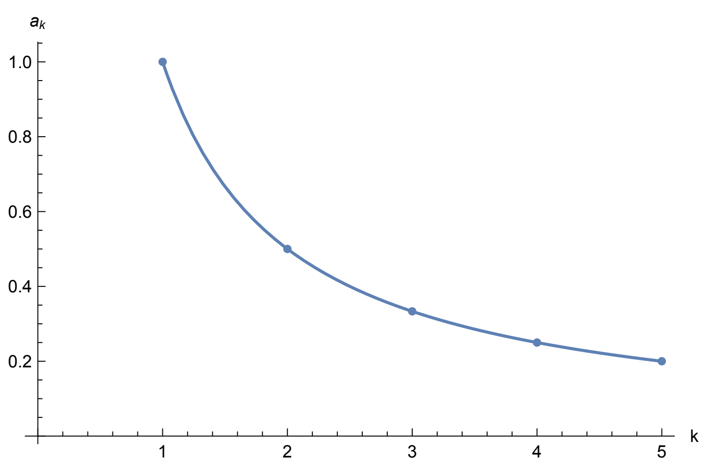

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- Approximate $\int_1^5 \frac{1}{x}\,dx$ using $n = 4$ and the *left* hand endpoints.
- Is that an underestimate, overestimate, or can't tell?
- How is that approximation related to $\sum_{k=1}^{4} \frac{1}{k}$?
- What is $\int_1^\infty \frac{1}{x}\,dx$? What can you conclude about $\sum_{k=1}^{\infty} \frac{1}{k}$?

Integral test

Given the infinite series $\sum_{k=a}^{\infty} a_k$, if there is an integrable function $f(x)$ such that

- $f(k) = a_k$ for $k \geq b$,
- $f(x) \geq 0$ for $x \geq b$,
- $f(x)$ is a decreasing function for $x \geq b$.

then the infinite series $\sum_{k=b}^{\infty} a_k$ converges if and only if the improper integral $\int_b^\infty f(x)\,dx$ converges.

Question

Use the integral test to determine if $\sum_{k=1}^{\infty} e^{-k}$ converges or not.

Question

For which values of p does the infinite series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converge and for which values of p does it diverge?

Error estimate

Once you know a series converges, you can approximate it with a partial sum.

$$\sum_{k=1}^{\infty} a_k \approx \sum_{k=1}^{n} a_k$$

Questions

- How do we compute the error in an approximation exact ≈ approx?
- What is the error in $\sum_{k=1}^{\infty} a_k \approx \sum_{k=1}^{n} a_k$?
- Write out term by term $\sum_{k=1}^{\infty} a_k$ and $S_3 = \sum_{k=1}^{3} a_k$ and find the error.
- In general what is a nice formula for the error in $\sum_{k=1}^{\infty} a_k \approx \sum_{k=1}^{n} a_k$?
- If the integral test applies, then we compare sums to areas under curves. What is a nice error estimate for the error here?

Questions

Consider the series $\Sigma_{k=1}^{\infty} \frac{1}{k^3}$. We know this converges by the integral test.

- What is the error in approximating $\Sigma_{k=1}^{\infty} \frac{1}{k^3}$ with $\Sigma_{k=1}^{5} \frac{1}{k^3}$?
- How should I choose n to approximate $\Sigma_{k=1}^{\infty} \frac{1}{k^3}$ with $\Sigma_{k=1}^{n} \frac{1}{k^3}$ so that the error is no more than 0.0001