Section 7.1: Integration By Parts Worksheet

Integration by parts

Let's say you don't like the integral $\int f(x)\, g'(x)\, dx$. You can rewrite it as

$$f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

if you like, and maybe that one will look better to you (one integral for another).

Note that this works for definite integrals, too: we simply add limits:

$$\int_a^b f(x)\, g'(x)\, dx = f(x)\, g(x) \Big|_a^b - \int_a^b f'(x)\, g(x)\, dx$$

So you've got choices. The first thing to look for in the integrand is a **product,** of two functions: one you wouldn't mind differentiating (*f(x)*), and the other you wouldn't mind anti-differentiating (think of it as *g'(x)*).

Alternate form

If we write *u=f(x)* and *dv=g'(x)dx*, then *du=f'(x)dx* and *v=g(x);* and if we can identify an integral as $\int u\, dv$, then we can rewrite it as $u\, v - \int v\, du$.

This makes it all look a little like a double substitution. It's actually just a good shorthand. I personally prefer the first form we considered, above -- but you're welcome to use this alternative form (and sometimes I do!).

Questions to submit

1. To evaluate $\int x^n \ln(x)\, dx$, use integration by parts with $f(x) = \ln(x)$ and $g'(x) = x^n$.
2. Using your results from problem one, what is $\int \ln(x)\, dx$?
3. Evaluate $\int_0^\pi x^2\, e^{-4x}\, dx$ using integration by parts.
4. Evaluate $\int x^3 \cos(x^2)\, dx$ using integration by parts, but first make a substitution.
5. What is the area of the region bounded by $y = \sin^{-1}(x)$, the x-axis, and $x = 1/2$? (How did we find the anti-derivative of $\tan^{-1}$?)
6. What is the volume of the solid obtained by rotating about the x-axis the region bounded by $y = x\sqrt{\ln(x)}$ and the x-axis for $1 \le x \le e$. (Use problem one!)

7. In problem one above, there is one special case $n = -1$. Use integration by parts in this particular instance to get $\int x^{-1} \ln(x)\, dx = \text{stuff} - \int x^{-1} \ln(x)\, dx$. Solve this equation for $\int x^{-1} \ln(x)\, dx$ to finish evaluating the integral. (Can you think of how this becomes a general rule? What is special about the integrand, $x^{-1} \ln(x)$?)

8. To evaluate $\int e^x \cos(x)\, dx$, use integration by parts twice. (Be sure to choose u and dv the same way both times. If you choose $u = e^x$ the first time, be sure to choose $u = e^x$ the second time. Or, if you choose u = trig the first time, choose u = trig the second time.) Then employ what you did in problem 7 to finish evaluating the integral.