Weekly Assignment 5

1. Recursively defined sequence

a. Determine whether the *sequence* defined as follows is convergent or divergent.

$a_1 = 1$

$a_n = 4 - a_{n-1}$ for $n > 1$

b. What happens if the first term is $a_1 = 2$?

2. Geometric series

a. Find the value of b such that $\sum\limits_{k=1}^{\infty} \left(\frac{1}{1+b}\right)^k = 2$.

b. Express the repeating decimal number 0.467467467 …. as a ratio of integers by first writing it as a geometric sum.

c. Repeat the second part with the repeating decimal number 0.999 …. What's the surprise?

3. Partial sums

a. If the nth partial sum of series $\sum\limits_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \ldots$ is $S_n = 3 - \frac{n}{2^n}$. Does the series converge? If so, to what value?

b. Write the first four partial sums of $\sum\limits_{k=1}^{\infty} \left(\frac{k}{2^k} - \frac{k+1}{2^{k+1}}\right)$. What is the form of S_n? Does the series converge? If so, to what value?