Section 10.3: Polar Coordinates

Different coordinate systems

The Cartesian coordinates for a point in the plane is an address for where the point is located. Use the coordinates to determine how to get to the given point.

Example

Street map of Tulsa, Oklahoma.

Example

If you are at the origin, to get to the point (5, −8), travel 5 units in the positive x direction, then go 8 units in the negative y direction.

Example

Map of the United States.

Polar Coordinates

Polar coordinates is a different addressing scheme for points in the plane that give you information on how to travel from the origin to the given point. Polar coordinates take the form (r, θ) where
 θ represents the angle from the positive x-axis to head along;
 r is the distance to travel along that heading.
For example, $(r, \theta) = (4, \pi/6)$ is the point one can reach by starting at the origin and walking 4 units along the ray that is 30° north of the positive x-axis.

Questions

- Plot the point given by polar coordinates $(r, \theta) = (2, \pi)$.
- Plot the point given by polar coordinates $(r, \theta) = (3, -\pi/4)$.
- How does the point $(r, \theta) = (1, \pi/3)$ compare with $(r, \theta) = (1, \pi/3 + 2\pi)$?
- In general, how does (r, θ) and $(r, \theta + 2\pi)$ compare?
- What might a negative value for r mean? Plot $(r, \theta) = (-4, \pi/6)$.
- How does the point $(r, \theta) = (1, \pi/3)$ compare with $(r, \theta) = (-1, \pi/3 + \pi)$?

- In general, how does $(-r, \theta)$ and $(r, \theta + \pi)$ compare?

Polar and Cartesian coordinates

Every point in the plane has both Cartesian coordinates and polar coordinates. How are they related?

- Write x and y in terms of r and θ.
- Find Cartesian coordinates for the point with polar coordinates $(1, \pi/3)$.
- Write r and θ in terms of x and y.
- Find polar coordinates for the point with Cartesian coordinates $(2, 1)$.

Polar curves

Typically, polar curves are represented by equations of the form
$$r = f(\theta)$$
where θ varies over a range of values.

Question

What is the curve given by $r = 5, 0 \le \theta \le 2\pi$?

Question

Plot the polar curve given by $r = 2 - \cos(\theta)$ for $0 \le \theta \le 2\pi$ by making a table of values and plotting points

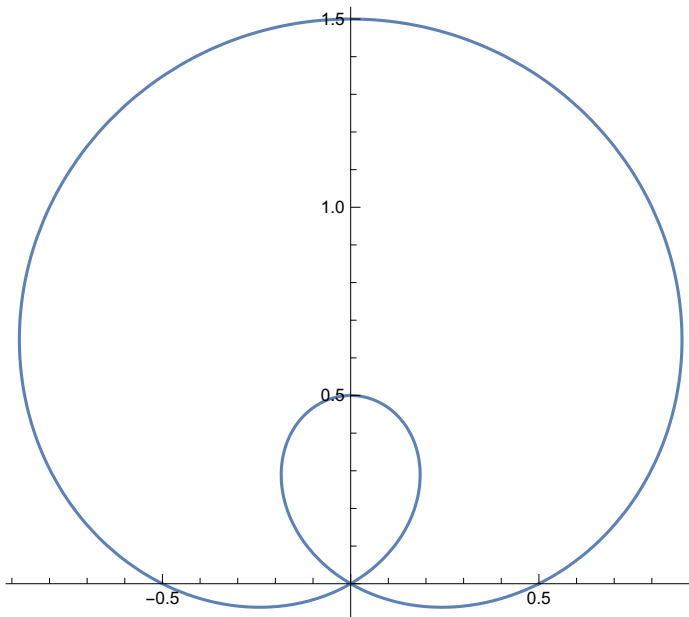| θ	$r = 2 - \cos(\theta)$
0	□
$\pi/8$	□
$\pi/4$	□
$3\pi/8$	□
$\pi/2$	□
$5\pi/8$	□
$3\pi/4$	□
$7\pi/8$	□
π	□
$9\pi/8$	□
$5\pi/4$	□
$11\pi/8$	□
$3\pi/2$	□
$13\pi/8$	□
$7\pi/4$	□
$15\pi/8$	□
2π	□

```
In[2]:= Manipulate[
    Show[Graphics[{Dashing[{0.01}], Line[{{0, 0}, (2 - Cos[θ]) {Cos[θ], Sin[θ]}}],
       Text[θ, .2 {Cos[θ/2], Sin[θ/2]}]}], PolarPlot[2 - Cos[a], {a, 0, θ}],
     PlotRange → {{-3, 3}, {-3, 3}}, Axes → True], {θ, 0.001, 4 π}]
```

Questions

- Identify the curve $r = 4\sin(\theta)$ by converting it to a Cartesian equation.
- Identify the curve $r = \frac{5}{\cos(\theta)}$ by converting it to a Cartesian equation.

Questions

The polar curve $r = \frac{1}{2} + \sin(\theta)$ is shown below.

- The origin is a point on this curve. What would be the r-value of this point?
- There are two angles associated with this point. What are they?

Question

The two polar curves $r = 1/2$ and $r = -\sin(\theta)$ are shown below. What are polar coordinates for their points of intersection?