Section 11.2 Series

Review

Questions

Which of the following sequences converge? To what do they converge?

- $\left\{\frac{2n+5}{3n-1}\right\}_{n=1}^{\infty}$
- $\left\{\frac{6^n}{11^n}\right\}_{n=0}^{\infty}$
- $\left\{(-1)^n \frac{3n^2+n-1}{n^2+5}\right\}_{n=0}^{\infty}$
- $\left\{\left(-\frac{5}{8}\right)^n\right\}_{n=2}^{\infty}$
- $\left\{\left(\frac{4}{3}\right)^n\right\}_{n=1}^{\infty}$

Monotonic sequences

Definition

A sequence that is either increasing or decreasing is said to be monotonic.

Technique 3

A bounded, monotonic sequence converges.

Questions

Consider the sequence $\left\{\frac{n+1}{n}\right\}$. Write out the first few terms of this sequence.

- Is this sequence monotonic?
- Is this sequence bounded?

Questions

Consider the sequence $\left\{\frac{n}{3^n}\right\}$. Write out the first few terms of this sequence.

- Is this sequence monotonic?
- Is this sequence bounded?

Infinite sums

Many quantities can be written as infinite sums or **series**.

Example

$$\pi = 3.14159265\ldots = 3 + \frac{1}{10} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{5}{10^4} + \frac{9}{10^5} + \frac{2}{10^6} + \frac{6}{10^7} + \frac{5}{10^8} + \ldots$$

Questions

- What is the decimal version of $\frac{1}{3}$?
- Write $\frac{1}{3}$ as an infinite sum.
- Write $\frac{1}{3}$ as an infinite sum using summation notation.
- As you go farther out in the sum what is happening to the individual terms?

Questions

- What is the value of the infinite sum $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots$?
- Write this infinite sum using summation notation.
- As you go farther out in the sum what is happening to the individual terms?

Partial sums

The partial sums for the series $a_1 + a_2 + a_3 + a_4 + \ldots$ are

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$S_4 = a_1 + a_2 + a_3 + a_4$$
$$\vdots$$

Questions

- What are the first 4 partial sums for the infinite series for
$$\pi = 3 + \frac{1}{10} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{5}{10^4} + \frac{9}{10^5} + \frac{2}{10^6} + \frac{6}{10^7} + \frac{5}{10^8} + \ldots?$$
- What do these partial sums represent?

Question

Write the k^{th} partial sum for $a_1 + a_2 + a_3 + a_4 + \ldots$ using the summation notation.

Definition

The infinite sum $\sum_{k=1}^{\infty} a_k$ converges if and only if its partial sums converge. If it converges, its value is the limit of the partial sums, $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$.

Question

Consider the series $\sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{k+1}\right)$.

- What are the first four partial sums for this series?
- What is the value of the n^{th} partial sum?
- Does this series converge? If so, to what value?

Geometric series

Definition

A geometric series has the form $\sum_{k=n_0}^{\infty} a\, r^k$ for some numbers n_0, a, and r.

Questions

Which of the following are geometric series?

- $\sum_{k=0}^{\infty} 2\left(\frac{1}{3}\right)^k$
- $\frac{7}{2} + \frac{7}{4} + \frac{7}{8} + \frac{7}{16} + \frac{7}{32} + \frac{7}{64} + \ldots$
- $\sum_{k=1}^{\infty} 2^{-k}\, 3^k$

Questions

Consider the geometric series $\sum_{k=0}^{\infty} r^k$. Let $S_n = \sum_{k=0}^{n-1} r^k = 1 + r + r^2 + \ldots + r^{n-1}$ be the n^{th} partial sum for this series.

- What is $r\, S_n$?
- What is $S_n - r\, S_n$?
- Solve $S_n - r\, S_n =$ (what you got above) for S_n and determine for what values of r the partial sum converges.
- Which of the geometric series converge, and for those that do converge to what value?
  - $\sum_{k=0}^{\infty} 2\left(\frac{1}{3}\right)^k$
  - $\frac{7}{2} + \frac{7}{4} + \frac{7}{8} + \frac{7}{16} + \frac{7}{32} + \frac{7}{64} + \ldots$

- $\sum_{k=1}^{\infty} 2^{-k} 3^{k}$

Questions

- Consider the geometric sum $\sum_{n=0}^{\infty} 4 \left(\frac{5}{6}\right)^{n}$
  - What are the first few terms of this sum?
  - What is the value of this infinite sum?
- Consider the geometric sum $\sum_{n=2}^{\infty} 3 \left(-\frac{3}{5}\right)^{n}$
  - What are the first few terms of this sum?
  - Rewrite this sum in the form $\sum_{n=0}^{\infty} a\, r^{n}$. What is a? What is r?
  - What is the value of this infinite sum?
- Consider the sum $\sum_{n=0}^{\infty} \frac{2^{n}+5^{n}}{7^{n}}$.
  - What are the first few terms of this sum?
  - Rewrite this infinite sum as the sum of two geometric series
  - What is the value of this infinite sum?

Question

Write the repeating decimal 0.121212121212… as a geometric series and find its value as a fraction.

Divergence Theorem

Questions

Which of the following sums converge? What is true about what happens to the individual terms of the sums?

- $\sum_{n=1}^{\infty} 2 = 2 + 2 + 2 + 2 + \ldots$
- $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$
- $\sum_{k=1}^{\infty} \frac{k}{k+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots$

Questions

Suppose the series $a_1 + a_2 + a_3 + a_4 + \ldots.$ converges to L.

- What is $S_{n+1} - S_n$?
- What is $\lim_{n\to\infty} S_{n+1}$? What is $\lim_{n\to\infty} S_n$? What is $\lim_{n\to\infty} S_{n+1} - S_n$?

Theorem

If $\lim_{k\to\infty} a_k \neq 0$, then the infinite sum $\sum_{k}^{\infty} a_k$ cannot converge.

Question

How do I know that $\sum_{k=1}^{\infty} \frac{k}{2k+1}$ diverges?