Section 12.1: 3D Coordinates

2D Coordinates

For points in the plane we have Cartesian coordinates (x, y) and polar coordinates (r, θ). Two numbers are needed to address any point.

Question

How are locations on earth's surface typically represented?

In[1526]:=
```
GeoPosition[Entity["City", {"Cincinnati", "Ohio", "UnitedStates"}]]
```

3D Coordinates

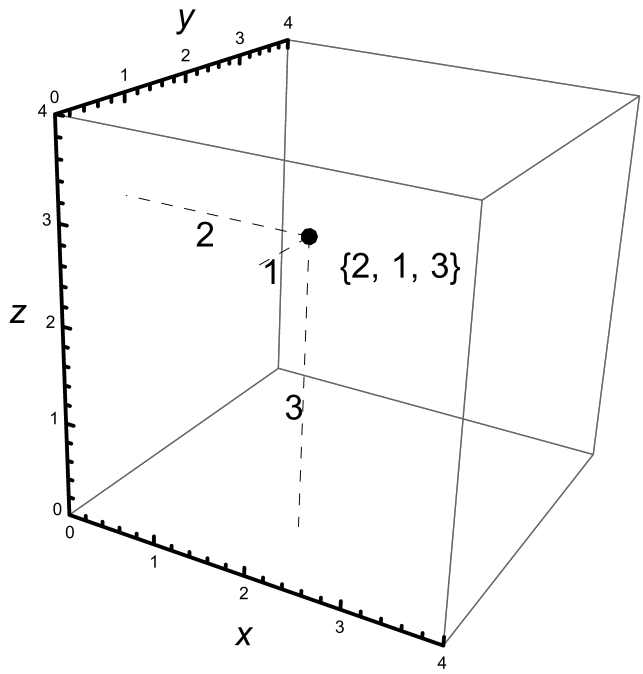
Question

What information is needed to locate the position of a flying plane?

Cartesian 3D coordinates

Start with the x-y plane. Add depth with the z-axis coming out from the plane at an angle of 90°. A point in space has coordinates (x, y, z) where

- z is the distance of the point from the x-y plane
- y is the distance of the point from the x-z plane
- x is the distance of the point from the y-z plane



Questions

- The equation $z = 3$ is the set of points (x, y, z) with $z = 3$. What is the shape of this set?
- The equation $x = 2$ is the set of points (x, y, z) with $x = 2$. What is the shape of this set?
- Give an equation for the plane that is parallel to the y-z plane and is 5 units from it in the positive x direction.
- Sketch the equation $x + y = 3$.
- The equation $y > 1$ is the set of points (x, y, z) with $y > 1$. What is the shape of this set?

Distance

Questions

We want to find the distance between (1, 2, 0) and (2, 1, 3).

- Draw a box with one corner at (1, 2, 0) and the diagonal corner at (2, 1, 3).
- What are the dimensions of this box?
- What is the distance between the two points?

Questions

- Repeat the above to find a distance formula for the distance between points (x_0, y_0, z_0) and (x_1, y_1, z_1).

- How does this formula compare with the one for the distance between two points in the plane?

Questions

Consider the triangle whose vertices are (3, −2, −3), (7, 0, 1), and (1, 2, 1).

- Is it a right triangle?
- Is it an isosceles triangle?

Questions

Consider the set of points (x, y, z) that are a distance of 2 from the origin (0, 0, 0).

- What is an equation that x, y, z must satisfy for (x, y, z) to be in this set?
- What is this shape?
- Generalize this to find an equation for any of this shape.

Questions

- The equation $(x-2)^2 + y^2 + (z+3)^2 = 4$ represents a sphere.
  - What is its center?
  - What is its radius?
  - Describe its intersections with each of the coordinate planes.
- The equation $x^2 + y^2 + z^2 + 2x - 4y - 10z = 0$ represents a sphere.
  - What is its center?
  - What is its radius?