

# Section 6.2: Exponential Functions

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## Exponential functions

An exponential function has the form  $f(x) = a(b)^x$  where  $a$  and  $b$  are some constants. The number  $b$  is called the *base*.

### Examples

- Social media: you develop the latest and greatest social media app. If you introduce it to 2 people, both of whom introduce it to 5 new people after one week, each of whom introduces it to 5 new people after week two, etc., how many people are using it after 10 weeks?
- Disease: the spread of disease can often be modeled using exponential functions. It is similar to the social media effect.
- Forensic science: heat loss can be reasonably modeled using exponential functions. In particular, if the ambient temperature stays mostly constant, then the temperature  $T$  of a body that is cooling at time  $t$  is approximated by the function  $T(t) = a + b \cdot c^t$  where  $a$  is the ambient temperature with  $b$  and  $c$  being constants that depend on the original temperature of the body and how fast it loses heat. One tool in the CSI toolbox for determining time of death is based on this model.
- Finance: determining the value of an investment is based on exponential functions.

### Question

Get together with someone who has a graphing calculator. Graph  $y = b^x$  for various values of  $b$ . Form some conjectures about what is true for different values of  $b$ .

### Question

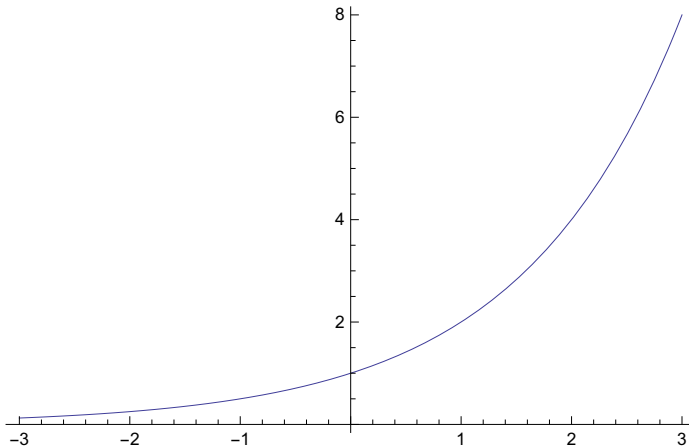
What are equivalent expressions for

- $b^0$
- $b^{-1}$
- $b^{x+y}$
- $b^{x-y}$
- $(b^x)^y$
- $b^{1/n}$

## Derivatives of exponential functions

The graph  $y = 2^x$  is

`Plot[2x, {x, -3, 3}]`



### Questions

- Does the power rule apply to the derivative of  $2^x$ ? Why or why not?
- What is the limit definition of the derivative for any function  $f(x)$ ?

### Question

- Apply the definition to  $f(x) = 2^x$  and factor as much as possible.
- Apply the definition to  $f(x) = 3^x$  and factor as much as possible.

### Natural exponential function

There exists a number between 2 and 3, Euler's number  $e$

`N[E, 100]`

such that  $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$  so that  $\frac{d}{dx}(e^x) = e^x$ .

### Questions

- What is the derivative of  $5x^2 + \cos(3x) - 4e^x$ ?
- Find an equation for the tangent line to  $y = e^{2x}$  at  $x = 0$ .
- What is the absolute maximum and the absolute minimum values of  $g(x) = 3e^{-x^2-6x}$  for  $-5 \leq x \leq 5$ ?
- Compute the area of the region bounded by  $y = e^x$  and  $y = x$  for  $0 \leq x \leq 2$ .

- What is the volume of the solid of revolution obtained by rotating  $y = e^x$  about the  $x$ -axis for  $-1 \leq x \leq 1$ ?
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## Homework

- Imath problems on exponential functions.