# Section 6.3: Logarithm Functions

## Review

The natural exponential function is the function $f(x) = e^x$ where $e \approx 2.71828\ldots$ It is natural in the calculus sense that $\frac{d}{dx} e^x = e^x$.

### Questions

- What does the graph of $y = e^x$ look like?
- What is the domain of $f(x) = e^x$?
- What is the range of $f(x) = e^x$?
- What are the following limits?
  - $\lim_{x \to \infty} e^x$
  - $\lim_{x \to -\infty} e^x$

### Questions

One mathematical model for how rumors spread is based on the function

$$p(t) = \frac{1}{1 + a\, e^{-kt}}$$

where $p(t)$ is the proportion of the population that knows the rumor at time $t$ and $a$, $k$ are positive constants.

- What is $\lim_{t \to \infty} p(t)$ and what does it mean for the rumor?
- Find the rate of the spread of the rumor.

### Question

- The bell of a trumpet is a surface of revolution obtained by rotating an exponential curve about the horizontal axis. Find the volume of the region bounded by $y = e^x$, $x = 0$, $x = 1$, $y = 0$ rotated about the $x$-axis.

## Properties of the natural exponential function

### Questions

Let $f(x) = e^x$.

- What is the domain and range of $f(x)$?
- What is $f(0)$?
- What is $\lim_{x\to\infty} f(x)$? What is $\lim_{x\to-\infty} f(x)$?
- Is $f(x)$ a one-to-one function?
- Does it have an inverse function?

# The natural logarithm function

Since $f(x) = e^x$ is a one-to-one function, it has an inverse function. That function is called the *natural logarithm* function and is denoted

$$f^{-1}(x) = \ln(x)$$

Everything we know about $\ln(x)$ comes from the fact that

$$y = \ln(x) \longleftrightarrow x = e^y$$

## Question

- What are some properties of inverse functions?

## Questions

- What is the domain and range of $\ln(x)$?
- What is $\ln(1)$?
- Graph $y = \ln(x)$
- What is $\lim_{x\to\infty} \ln(x)$? What is $\lim_{x\to 0^+} \ln(x)$?
- What is the solution to $e^{2x-1} = 5$?

# Properties of logarithms

In general $f(x) = a^x$ is a one-to-one function as long as base $a$ is a positive number other than 1. Since it is one-to-one, it has an inverse and that inverse is denoted

$$f^{-1}(x) = \log_a(x)$$

This means $y = \log_a(x)$ and $x = a^y$ are equivalent.

## Questions

- What is $a^{\log_a(x)}$?
- What is $\log_a(a^x)$?

Properties of logarithms come directly from properties of exponentials. For example,

$$a^x\, a^y = a^{x+y} \longleftrightarrow \log_a(u\, v) = \log_a(u) + \log_a(v)$$

## Question

What other exponent properties are there? What logarithm properties do they lead to?

Since the natural base is the best for calculus, other exponentials are often rewritten in terms of $e$ if any calculus is to be done.

## Example

Using the fact that $e^x$ and $\ln(x)$ are inverse, $2^x = e^{\ln(2^x)} = e^{x \ln(2)}$. $\ln(2)$ is just a number

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Log[2.]
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## Questions

- What is $\frac{d}{dx} 2^x$?
- Find an equation for the tangent line to $y = 5^x$ at $x = 1$.
- What is the area of the region bounded by $y = 10^x$ and the $x$-axis for $-1 \le x \le 1$?

# Homeworks (in progress)

- Weekly assignment 1.
- IMath problems on logarithmic functions.
- IMath problems on exponential function.