# Section 6.4: Derivatives of Logarithm Functions

## Review

### Logarithms

Since $f(x) = e^x$ is a one-to-one function, it has an inverse function. That function is called the *natural logarithm* function and is denoted

$$f^{-1}(x) = \ln(x)$$

Everything we know about $\ln(x)$ comes from the fact that

$$y = \ln(x) \longleftrightarrow x = e^y$$

In general $f(x) = a^x$ is a one-to-one function as long as base $a$ is a positive number other than 1. Since it is one-to-one, it has an inverse and that inverse is denoted

$$f^{-1}(x) = \log_a(x)$$

This means $y = \log_a(x)$ and $x = a^y$ are equivalent.

### Questions

Let $g(x) = 2\,e^x - 5\,x$.

- Find all critical numbers for $g(x)$. Classify each as either a local maximum, a local minimum, or neither.
- Determine where $y = g(x)$ is concave up and where it is concave down.

### Question

Let $h(x) = x\,e^{x/3}$.

- Find the absolute maximum value and the absolute minimum value of $h(x)$ for $-2 \le x \le 2$.

### Question (a little challenging...)

- What is an equivalent way to write $g(x) = (x^2 + 1)^{\cos(x)}$? What is its domain?

## Derivative of $\ln(x)$

Let $y = \ln(x)$, then $x = e^y$. Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{d}{dx}(x) = \frac{d}{dx}(e^y)$$

$$\longrightarrow 1 = e^y \frac{dy}{dx}$$

$$\longrightarrow \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

The derivative of $\ln(x)$ is

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

## Questions

- Find any maximum or minimum points to $f(x) = x \ln(x)$.
- What is an equation for the tangent line to $g(x) = x^{\sin(x)}$ when $x = \pi/2$?
- Let $h(x) = \ln(-x)$. What is its domain? What is its derivative?
- What is $\int x^{-1}\, dx$ ?
- Find the area of the region bounded by the $x$-axis and $y = \frac{1}{x}$ for $1 \le x \le 2$.
- Find the area of the region bounded by the $x$-axis and $y = \frac{1}{x}$ for $-3 \le x \le -1$.
- Find the volume of the solid of revolution obtained by rotating the region bounded by the $x$-axis and $y = 1 + \frac{1}{x}$ for $1/2 \le x \le 2$.

# Homework (in progress)

- Weekly assignment 1 is coming due soon. **Keep up with your homework assignment page for dates!**
- Imath problems on differentiating logarithmic functions.
- Imath problems on logarithmic functions.
- Imath problems on exponential functions.