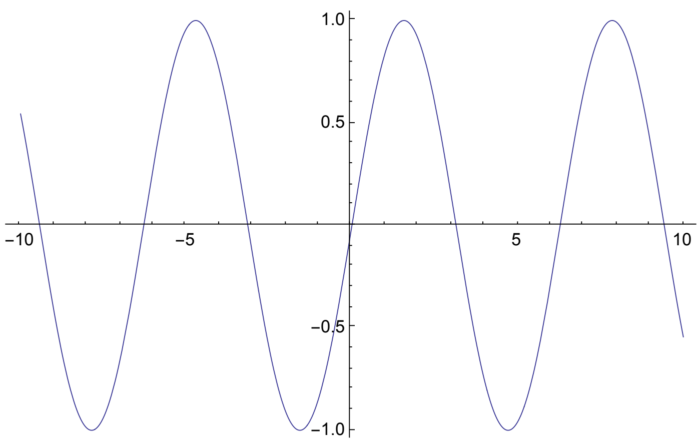
Section 6.6: Inverse Trigonometric Functions

Sine function

The graph of the sine function $f(x) = \sin(x)$ is



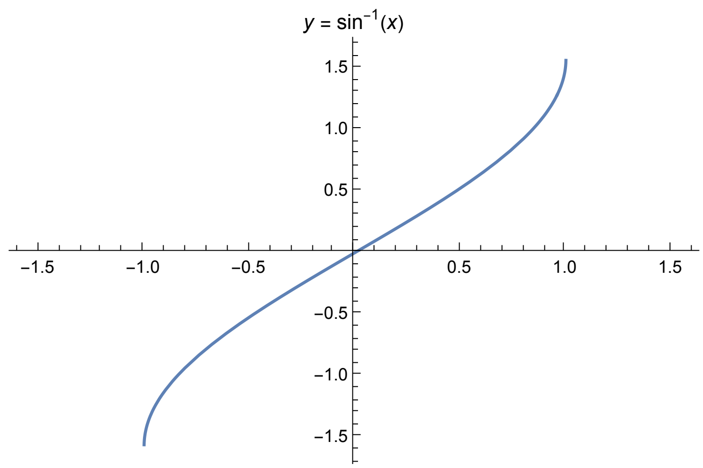
Questions

- Solve $\sin(x) = 0$ for x.
- Solve $\sin(x) = 1$ for x.
- Solve $\sin(x) = 2$ for x.
- What is the domain and range of $\sin(x)$?
- Is $\sin(x)$ a one-to-one function over its entire domain?

Inverse sine

The function $f(x) = \sin(x)$ for $-\pi/2 \leq x \leq \pi/2$ is a one-to-one function and has an inverse called the inverse sine or arc sine function

$$f^{-1}(x) = \sin^{-1}(x) = \arcsin(x)$$



Questions

- What are the domain and range of $\sin^{-1}(x)$?
- What is $\sin^{-1}(1)$?
- What is $\sin(\sin^{-1}(0.2))$?
- For what values of x does $\sin^{-1}(\sin(x)) = x$?
- What is $\sin^{-1}(\sin(2\pi))$?
- Sketch the graph $y = \sin^{-1}(x)$.
- Solve $4\sin^{-1}(3x) = \pi$ for x.

Derivative of inverse sine

To find the derivative of $\sin^{-1}(x)$, set $y = \sin^{-1}(x)$. This is equivalent to

$$\sin(y) = x$$

Find $\frac{dy}{dx}$ using implicit differentiation.

$$\frac{d}{dx}\sin(y) = \frac{d}{dx}x$$

$$\longrightarrow \cos(y)\frac{dy}{dx} = 1$$

$$\longrightarrow \frac{dy}{dx} = \frac{1}{\cos(y)}$$

We need the answer in terms of x, not y. Since $y = \sin^{-1}(x)$, we could write the derivative as

$$\frac{dy}{dx} = \frac{1}{\cos(\sin^{-1}(x))}$$

but there is a preferred form. Since $-\pi/2 \le y \le \pi/2$, angle y is in the first or fourth quadrant; $\cos(y) \ge 0$. Using this along with the Pythagorean identity

$$\cos^2(y) + \sin^2(y) = 1$$

to write $\cos(y)$ in terms of $\sin(y)$.

Questions

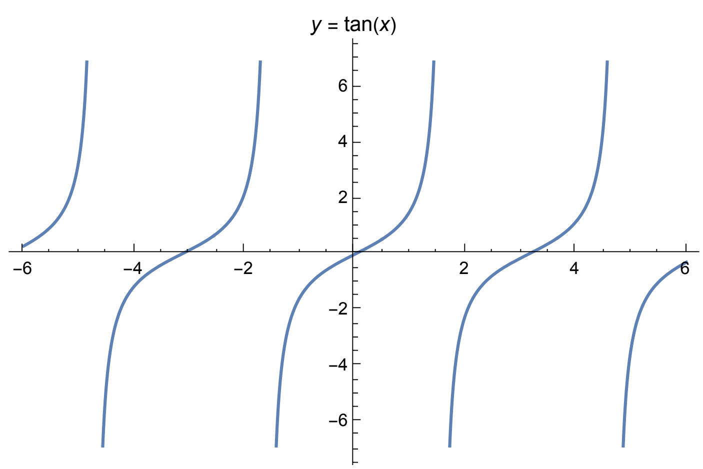
- Write cos(y) in terms of sin(y) if $-\pi/2 \le y \le \pi/2$.
- Using $x = \sin(y)$, write $\frac{dy}{dx} = \frac{1}{\cos(y)}$ in terms of x.
- Find an equation for the tangent line to $y = x \sin^{-1}(2x)$ at $x = 1/2$.

Questions

- Let $g(x) = \sin^{-1}(x/2) - x$.
  - What is the domain of $g(x)$?
  - Find the maximum and minimum values of $g(x)$.
  - What is the range of $g(x)$?
- Find the area bounded by $y = \frac{1}{\sqrt{1-x^2}}$, the x-axis, and $x = \pm 1/2$.
- Evaluate the definite integral $\int_{\ln(1/2)}^{\ln\left(1/\sqrt{2}\right)} \frac{e^t}{\sqrt{1-e^{2t}}}\,dt$

Inverse tangent

The graph of $y = \tan(x)$ is



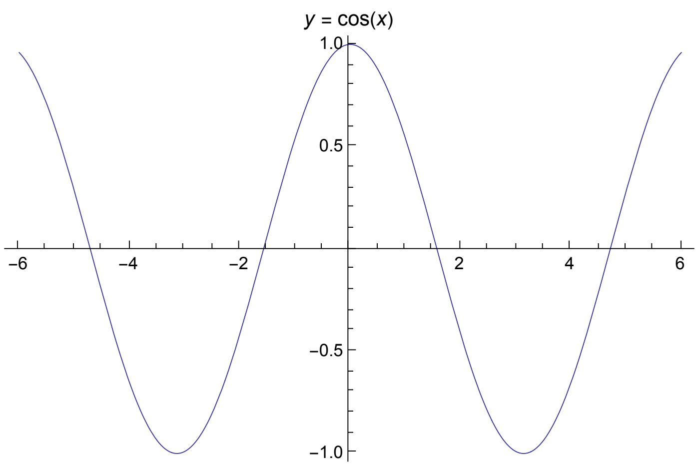
Questions

- What are the domain and range of tan(x)?
- What is $\lim_{x \to \pi/2^-} \tan(x)$?
- What do you think is the standard restricted domain used to define an inverse tangent function, $\tan^{-1}(x)$ or arctan(x)?
- Find the solutions closest to 0 to the equation $3\tan^3(x) + 1 = 0$.

- Since tan and $\tan^{-1}$ are inverse functions, $y = \tan^{-1}(x) \longleftrightarrow \tan(y) = x$. Use this along with implicit differentiation to determine the derivative of $\tan^{-1}(x)$, $\frac{dy}{dx} = ?$
- What is the new antidifferentiation formula we can get from this derivative?

Inverse cosine

The graph of $y = \cos(x)$ is

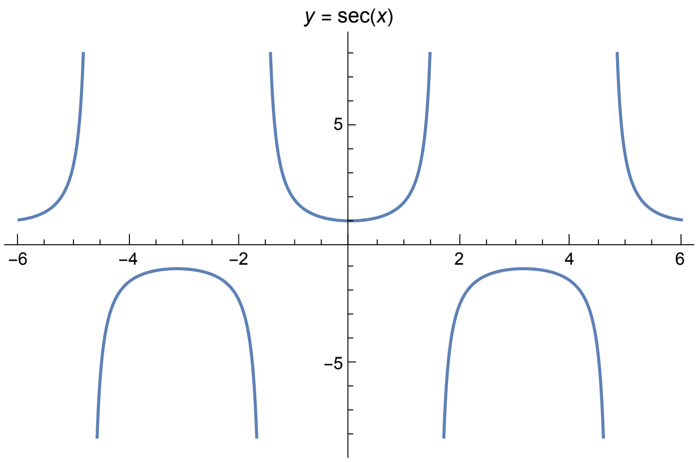


Questions

- What are the domain and range of $\cos(x)$?
- What do you think is the standard restricted domain used to define an inverse cosine function, $\cos^{-1}(x)$ or $\arccos(x)$?
- Since cos and $\cos^{-1}$ are inverse functions, $y = \cos^{-1}(x) \longleftrightarrow \cos(y) = x$. Use this along with implicit differentiation to determine the derivative of $\cos^{-1}(x)$, $\frac{dy}{dx} = ?$
- How does this compare to the derivative of $\sin^{-1}(x)$?

Inverse secant

The graph of $y = \sec(x)$ is



Questions

- What are the domain and range of sec(x)?
- There are two “standard” restricted domain used to define an inverse secant function, $\sec^{-1}(x)$ or arcsec(x). What do you think they are?
- Since sec and $\sec^{-1}$ are inverse functions, $y = \sec^{-1}(x) \longleftrightarrow \sec(y) = x$. Use this along with implicit differentiation to determine the derivative of $\sec^{-1}(x)$: $\frac{dy}{dx} = ?$