Section 7.1: Integration By Parts

Integration techniques

Antidifferentiation is the reverse of differentiation. Anything we can learn about antidifferentiation must come from differentiation.

- Power rule: $\frac{d}{dx}(x^n) = n\,x^{n-1} \longrightarrow \int x^k\,dx = \frac{1}{k+1} x^{k+1} + C$ if $k \neq -1$.
- Chain rule: $\frac{d}{dx}(F(u(x))) = F'(u(x))\,u'(x) \longrightarrow \int f(u(x))\,u'(x)\,dx = \int f(u)\,du$

Questions

- What is $\int \frac{1}{1+x^2}\,dx$?
- What is $\int \sin(2\,x)\,e^{\cos(2\,x)}\,dx$?

Questions

- What does the product rule say about $(u(x)\,v(x))'$.
- Antidifferentiate what you obtained.

Integration by parts

If you can identify $u(x)$ and $dv(x)$ in an integral, try rewriting the integral as

$$\int u\,dv = u\,v - \int v\,du$$

Example

To evaluate $\int x\,e^x\,dx$ let's try integration by parts. Identify u and dv.

$$u = x$$
$$dv = e^x\,dx$$

This leads to

$$du = dx$$
$$v = \int dv = \int e^x\,dx = e^x$$

Only one antiderivative is needed; any value of C at this point in "$+C$" works. Applying the integration by parts formula $\int u\,dv = u\,v - \int v\,du$ produces

$$\int x\,e^x\,dx = x\,e^x - \int e^x\,dx$$

The new integral is one of our basic integrals. The final answer is

$$\int x\, e^x\, dx = x\, e^x - e^x + C$$

Questions

Consider the integral $\int x \cos(2\,x)\, dx$.

- What is $\int x \cos(2\,x)\, dx$ if you use integration by parts with $u = x$ and $dv = \cos(2\,x)\, dx$?
- What would happen if you try to use integration by parts with $u = \cos(2\,x)$ and $dv = x\, dx$?

Question

How can you use integration by parts to evaluate $\int x^2 \sin(3\,x)\, dx$?

Question

How can you use integration by parts to evaluate $\int \tan^{-1}(x)\, dx$?