Section 7.2: Trigonometric Integration

Review

We have three antidifferentiation techniques:

1. Recognize it immediately as the derivative of some function, for example
   $\int e^x \, dx = e^x + C$
2. Try substituting for some internal part. Don't forget the differential. For example, to evaluate $\int x^2 \cos(x^3) \, dx$ use the substitution $u = x^3$ and $du = 3x^2 \, dx$ to replace the above integral with the simpler one
   $\int \frac{1}{3} \cos(u) \, du = \frac{1}{3} \sin(u) + C = \frac{1}{3} \sin(x^3) + C$
3. Integration by parts:
   $\int u \, dv = u \, v - \int v \, du$

Questions

Choose an appropriate technique to evaluate the following integrals.

- Find the area of the region bounded above by $y = x \, e^{-x}$, below by the x-axis, and the line $x = 5$.
- Find the volume of the surface of revolution generated by rotating about the x-axis the region bounded above by $y = x \sec(x^3)$, below by the x-axis, and the line $x = 1$.

Trigonometric identities

Pythagorean identities

$\cos^2(\theta) + \sin^2(\theta) = 1$

Questions

- Divide this equation by $\cos^2(\theta)$ to get another Pythagorean identity.
- Divide this equation by $\sin^2(\theta)$ to get another Pythagorean identity.

Double angle formulas

The sine and cosine sum formulas are very important:

$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\sin(\beta)\cos(\alpha)$.

$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

Questions

- What is $\sin(2\,\theta)=\sin(\theta+\theta)$?
- What is $\cos(2\,\theta)=\cos(\theta+\theta)$?
- Rewrite this using a Pythagorean trigonometric identity so that $\cos(2\,\theta)$ is in terms of only $\cos(\theta)$.
- Rewrite this using a Pythagorean trigonometric identity so that $\cos(2\,\theta)$ is in terms of only $\sin(\theta)$.

Trigonometric integrals

Questions

What are the following derivatives?

- $\frac{d}{dx}\cos(x)$
- $\frac{d}{dx}\sin(x)$
- $\frac{d}{dx}\tan(x)$
- $\frac{d}{dx}\sec(x)$
- $\frac{d}{dx}\cot(x)$
- $\frac{d}{dx}\csc(x)$

Questions

- Evaluate $\int\cos^4(x)\sin(x)\,dx$ using the substitution $u=\cos(x)$.
- Evaluate $\int\sin^6(x)\cos^3(x)\,dx$ by writing this as
  $\int\sin^6(x)\cos^2(x)\cos(x)\,dx=\int\sin^6(x)\,(1-\sin^2(x))\cos(x)\,dx$.
- Find the area under the curve $y=\sin^3(x)\cos^3(x)$ for $0\le x\le\pi/2$.
- Evaluate $\int\cot^4(x)\csc^2(x)\,dx$.
- Evaluate $\int\tan^2(x)\sec^4(x)\,dx$.

Questions

- Since $\cos(2\,\theta)=2\cos^2(\theta)-1$, solve for $\cos^2(\theta)$ to get $\cos^2(\theta)=\frac{1+\cos(2\,\theta)}{2}$. Use this to evaluate
  $\int\cos^2(\theta)\,d\theta=\int\frac{1+\cos(2\,\theta)}{2}\,d\theta$.

- Find the volume of the solid of revolution obtained by rotating about the x-axis the region above the x-axis and one arch of the sine curve $y = \sin(x)$.