Review for Exam 2

Exam rules

- No books nor cell phones are allowed. You can have one 8.5×11 inch sheet of notes, both sides. You are allowed to have a calculator, but not one that can do calculus.
- You have the full period to work on it. When you are finished, you can give me your exam and leave.

Format

- Questions will be similar to daily homework questions and weekly assignment questions.
- There are six problems, some with parts. The total number of question parts is 11, each worth 9 points.
- You must show your work. For example, if you need to evaluate a definite integral, you must derive the antiderivative and express how you evaluate it.

Topics

- From calculus 1
  - Find area
  - Find volume for solids of revolution
- From exam 1 material you will need to be familiar with exponential, logarithmic, and inverse trigonometric functions and their derivatives.
- Section 7.1: Integration by parts (product rule backwards):
  - $\int f(x)\, g'(x)\, \mathrm{dx} = f(x)\, g(x) - \int f'(x)\, g(x)\, \mathrm{dx}$
  - $\int u\, dv = u\, v - \int v\, du$
  - Be familiar with the types of integrals for which this technique is useful.
    - $\int x^n\, e^{a\, x}\, dx$
    - $\int x^n \sin(a\, x)\, dx$
    - $\int x^n \cos(a\, x)\, dx$
    - $\int x^n \ln(x)\, dx$
    - $\int x^n$(inverse trig function) dx
- Section 7.2: Trigonometric integrals (and at least three trig identities)

- $\sin^2(x) + \cos^2(x) = 1$
- $\sin(x + y) = \sin(x)\cos(y) + \sin(y)\cos(x)$
- $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\int \sin^n(a\,x)\cos^m(a\,x)\,dx$ if at least one of n or m is an odd integer.
- $\int \sin^n(a\,x)\cos^m(a\,x)\,dx$ if both are even integers.
- $\int \tan^n(a\,x)\sec^m(a\,x)\,dx$ if m is even.
- $\int \tan^n(a\,x)\sec^m(a\,x)\,dx$ if n is odd.

- Section 7.3: Trigonometric substitution
  - If the integral involves $\sqrt{a^2 - x^2}$ use $x = a\sin(\theta)$ and $dx = a\cos(\theta)\,d\theta$.
  - If the integral involves $\sqrt{a^2 + x^2}$ use $x = a\tan(\theta)$ and $dx = a\sec^2(\theta)\,d\theta$.
  - If the integral involves $\sqrt{x^2 - a^2}$ use $x = a\sec(\theta)$ and $dx = a\sec(\theta)\tan(\theta)\,d\theta$.
- Section 7.7: Numerical integration:
  - Trapezoidal rule with error estimate (as the average of RRR and LRR)
  - Midpoint rule with error estimate
  - Simpson's rule with error estimate (as the weighted average of Midpoint and Trapezoidal)
- Section 7.8: Improper integrals.

Studying

- Try problems you haven't worked from the exercises from the corresponding section of your calculus book. For those problems that ask you to use a graphing calculator, on our exam I would give you the graph.
- Review the weekly assignments. Remember which ones caused you the most trouble. Find similar examples in the textbook and the posted outlines, then try similar exercises in the textbook.
- Visit either me or the Math/Stats tutoring lab in MEP 457 for walk-in tutoring by upper level math/stats students with your questions.

Sample questions

1. Choose an appropriate technique of integration to evaluate the following integrals.

1.1. $\int \cos^3(x)\sin^5(x)\,dx$

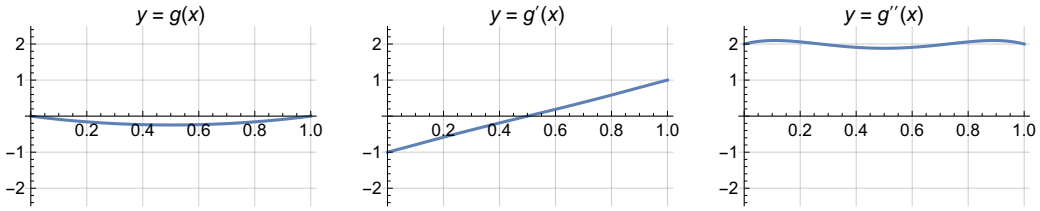
1.2. $\int x^2 e^{-x}\,dx$

1.3. $\int_0^1 \frac{\sqrt{x} - x}{x}\,dx$

1.4. $\int_0^1 x^3\sqrt{x^2 + 1}\,dx$ (remember: there's often more than one way to solve a problem!)

2. Consider $\int_0^1 \tan^{-1}(x^2 - x)\, dx$.

2.1. The plot of its graph and the graphs of its first two derivatives is shown below. Using the trapezoid rule error estimate, find a value of n for which the trapezoid rule is guaranteed to approximate the integral with error less than 0.01.



2.2. Using your value of n, approximate the integral with the trapezoid rule.

3. Find the volume of the solid obtained by rotating about the x-axis the region bounded by $y = \sqrt{x \ln(x)}$ and the x-axis for $1 \le x \le e$.

4. Consider the integral $\int \frac{x^2+1}{x^3+4x^2}\, dx$.

4.1. Use partial fraction decomposition to rewrite the integrand.

4.2. Evaluate the integral.