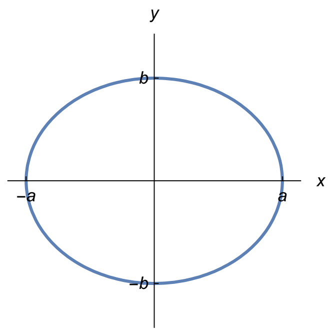
Section 7.3: Trigonometric Substitution

Review

- To evaluate an integral involving $\sqrt{a^2 - x^2}$ use the trigonometric substitution $x = a\sin(\theta)$. Then
$$\begin{aligned}\sqrt{a^2 - x^2} &= \sqrt{a^2 - a^2\sin^2(\theta)}\\ &= \sqrt{a^2(1 - \sin^2(\theta))} = \sqrt{a^2\cos^2(\theta)}\\ &= a\cos(\theta)\end{aligned}$$
- To evaluate an integral involving $\sqrt{a^2 + x^2}$ use the trigonometric substitution $x = a\tan(\theta)$. Then
$$\begin{aligned}\sqrt{a^2 + x^2} &= \sqrt{a^2 + a^2\tan^2(\theta)}\\ &= \sqrt{a^2(1 + \tan^2(\theta))} = \sqrt{a^2\sec^2(\theta)}\\ &= a\sec(\theta)\end{aligned}$$
- To evaluate an integral involving $\sqrt{x^2 - a^2}$ use the trigonometric substitution $x = a\sec(\theta)$. Then
$$\begin{aligned}\sqrt{x^2 - a^2} &= \sqrt{a^2\sec^2(\theta) - a^2}\\ &= \sqrt{a^2(\sec^2(\theta) - 1)} = \sqrt{a^2\tan^2(\theta)}\\ &= a\tan(\theta)\end{aligned}$$
- To evaluate an integral involving $\sqrt{b^2x^2 - a^2}$, $\sqrt{b^2x^2 + a^2}$, or $\sqrt{a^2 - b^2x^2}$. Then use $b\,x = a(\text{trig function})$ or $x = \frac{a}{b}(\text{trig function})$.

Problems to submit

1. Use trigonometric substitution to evaluate $\int\sqrt{4 - 9x^2}\,dx$.
2. Use trigonometric substitution to evaluate $\int_0^2 \frac{x^3}{\sqrt{2x^2+1}}\,dx$.
3. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



3.1. Solve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for y.

3.2. Find the area of this ellipse.

4. Trigonometric substitution can be useful even if no square roots are involved. Use it to evaluate $\int \frac{1}{(1+x^2)^2}\,dx$.

5. What is an appropriate trigonometric substitution to use in the integral $\int \sqrt{(2x-1)^2 + 25}\,dx$? You don't have to evaluate the integral, just give me the substitution along with reasons for your choice.