Simulating a Normal Distribution

Andy Long

Spring, 2020

- Suppose we want to simulate draws from a normal distribution:

Let's say with mean 2 and standard deviation 3, and we want to generate 1000 values from this distribution:

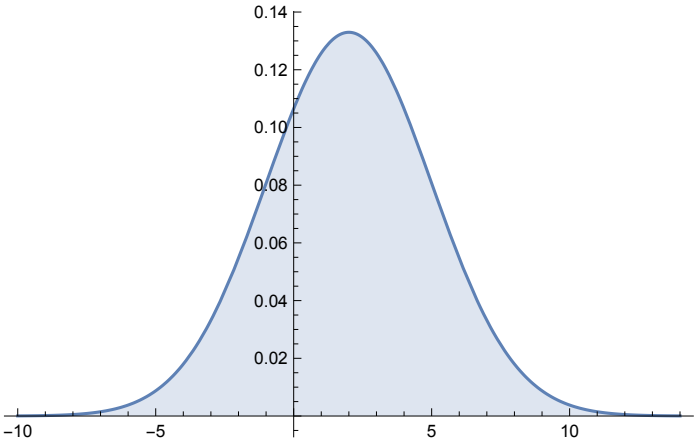
In[876]:=
```
mu = 2;
sigma = 3;
samplesize = 1000;
(* we'll look at stuff between plus and
 minus 4 standard deviations from the mean: *)
leftint = mu - 4 * sigma;
rightint = mu + 4 * sigma;
```

Just so you know, here are the Probability Density Function and Cumulative Distribution Function for this Normal (the cumulative is just the integral of the pdf, from -infinity to x):

In[881]:=
```
PDF[NormalDistribution[mu, sigma], x]
Plot[%, {x, leftint, rightint}, Filling → Axis]
CDF[NormalDistribution[mu, sigma], x]
Plot[%, {x, leftint, rightint}, Filling → Axis]
```

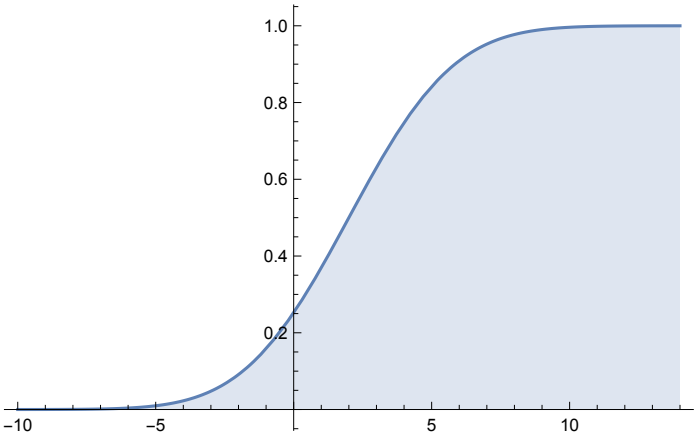
Out[881]=
$$\frac{e^{-\frac{1}{18}(-2+x)^2}}{3\sqrt{2\pi}}$$

Out[882]=



Out[883]=
$$\frac{1}{2}\,\text{Erfc}\left[\frac{2-x}{3\sqrt{2}}\right]$$

Out[884]=

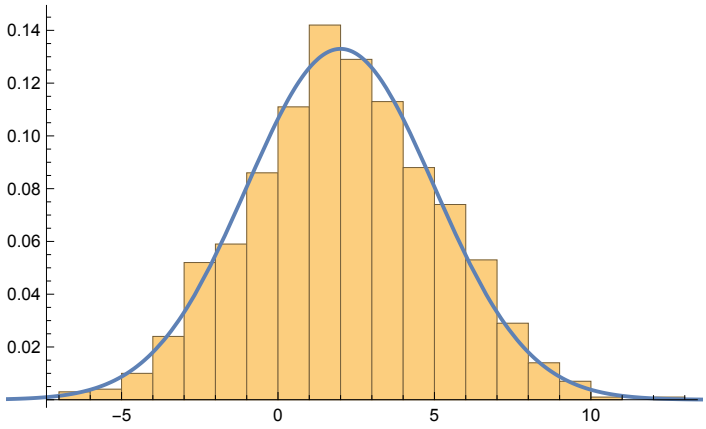


Well, Mathematica makes that easy, with RandomVariate,

so this is the thing that we're going to do "the hard way":

In[885]:=
```
randomData = RandomVariate[NormalDistribution[mu, sigma], samplesize];
Show[
 Histogram[randomData, 20, "ProbabilityDensity"],
 Plot[PDF[NormalDistribution[mu, sigma], x],
  {x, leftint, rightint}, PlotStyle → Thick]]
```

Out[886]=



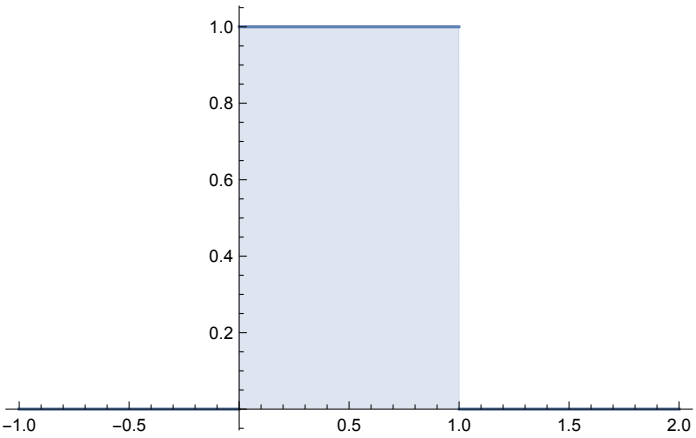
- We could do this ourselves, however, for any distribution. I'm just going to illustrate it here with this normal.

We start with a uniform probability density function (pdf), and pull a samplesize of random values from it:

In[887]:=
```
PDF[UniformDistribution[{0, 1}], x]
Plot[%, {x, -1, 2}, Filling → Axis]
us = RandomVariate[UniformDistribution[{0, 1}], samplesize];
```

Out[887]=
$$\begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Out[888]=



Now we solve u=normalcdf(0,1,z) for z, and then we have our x from the z: x = mu + sigma*z

In[896]:=
```
zs = Table[Solve[CDF[NormalDistribution[0, 1], z] == us[[i]]], {i, 1, samplesize}];
zs[[1]]
zs = Table[z /. zs[[i]][[1]], {i, 1, samplesize}];
zs[[1]]
xs = mu + sigma * zs;
xs[[1]]
```

Out[897]= $\{\{z \to 0.854882\}\}$

Out[899]= 0.854882

Out[901]= 4.56465

In[895]:=
```
Show[Histogram[xs, 20, "ProbabilityDensity"],
 Plot[PDF[NormalDistribution[mu, sigma], x],
  {x, leftint, rightint}, PlotStyle → Thick]]
```

Out[895]=