```
In[1333]:= f[x_] := x Log[x]
f'[x]
NewtonIteration[x_] := x - f[x] / f'[x]
NewtonIteration[x]
x0 = .5
x1 = NewtonIteration[x0]
x2 = NewtonIteration[x1]
x3 = NewtonIteration[x2]
```

Out[1334]= $1 + \text{Log}[x]$

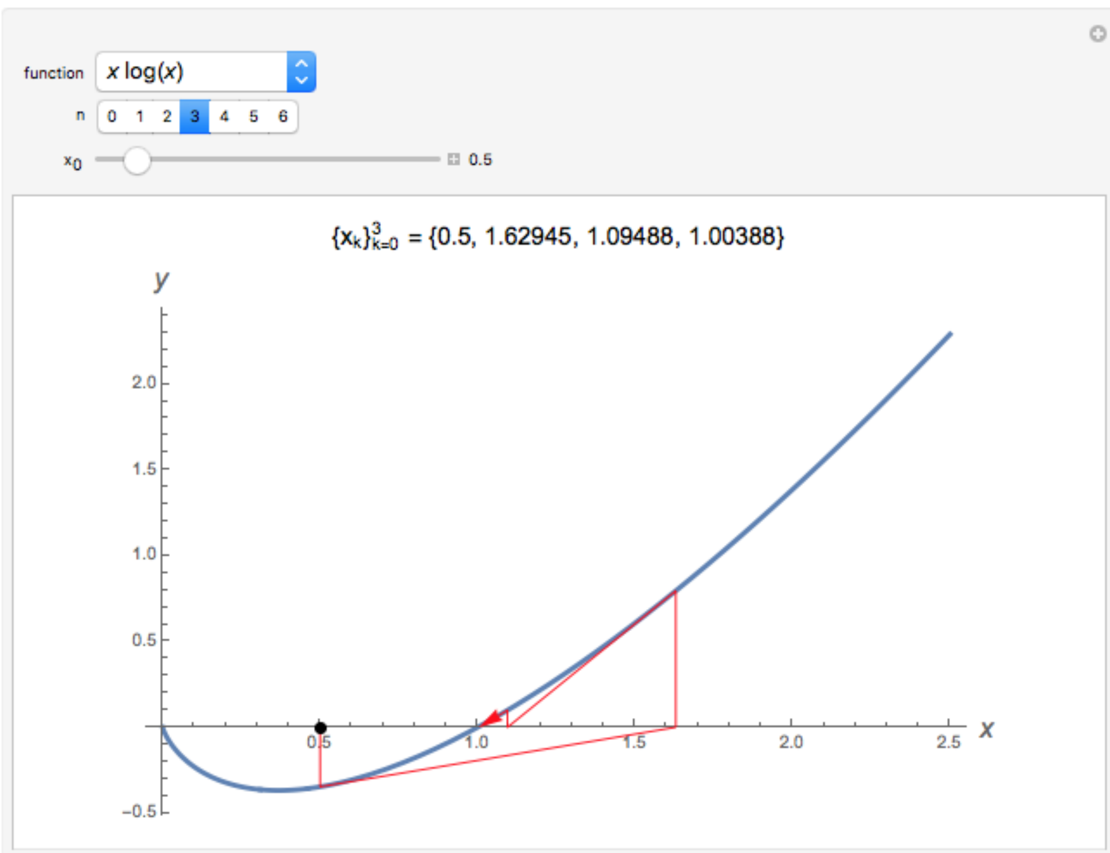
Out[1336]= $x - \dfrac{x\,\text{Log}[x]}{1 + \text{Log}[x]}$

Out[1337]= 0.5

Out[1338]= 1.62945

Out[1339]= 1.09488

Out[1340]= 1.00388



The model form was chosen by a+(0.49-a)E^(-b(t-8))
making it clear that after 8 weeks, t=8, so the exponential will have a zero argument -- and so the expression becomes .49.

a will be the horizontal asymptote (as t goes to infinity, the exponential will die and only the constant a will be left).
The exponential gives us a gradual slide to the asymptote, the slope of the slide determined by b.

In[1293]:=
```
xs = {8, 8, 10, 10, 10, 10, 12, 12, 12, 12, 14, 14, 14, 16, 16, 16, 18, 18, 20, 20, 20, 22,
   22, 22, 24, 24, 24, 26, 26, 26, 28, 28, 30, 30, 30, 32, 32, 34, 36, 36, 38, 38, 40, 42};
ys = {49, 49, 48, 47, 48, 47, 46, 46, 45, 43, 45, 43, 43, 44, 43, 43, 46, 45, 42, 42, 43, 41,
   41, 40, 42, 40, 40, 41, 40, 41, 41, 40, 40, 40, 38, 41, 40, 40, 41, 38, 40, 40, 39, 39};
ys = ys / 100;
data = Transpose[{xs, ys}];
```

In[1297]:=
```
lm = NonlinearModelFit[data, a + b * t, {a, b}, t]
lm = LinearModelFit[data, x, x]
lm["ParameterTable"]
lm["AdjustedRSquared"]
lm["ANOVATable"]
lm["ParameterConfidenceIntervals"]
```

Out[1297]= FittedModel[0.48551 − 0.00271679 t]

Out[1298]= FittedModel[0.48551 − 0.00271679 x]

Out[1299]=

	Estimate	Standard Error	t-Statistic	P-Value
1	0.48551	0.00589066	82.4204	4.44198×10^{-48}
x	−0.00271679	0.000243115	−11.1749	3.67471×10^{-14}

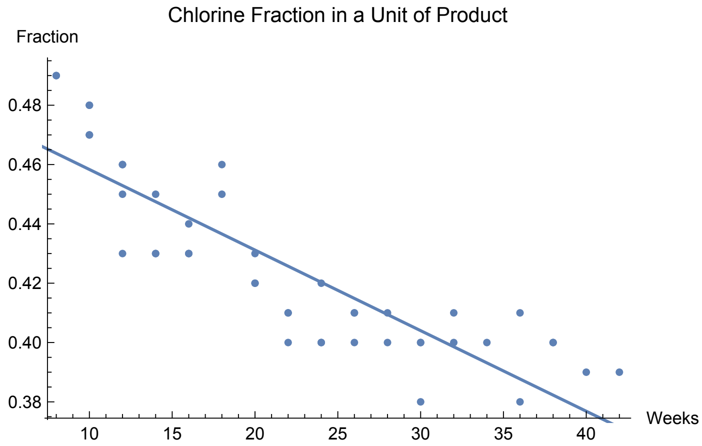
Out[1300]= 0.742328

Out[1301]=

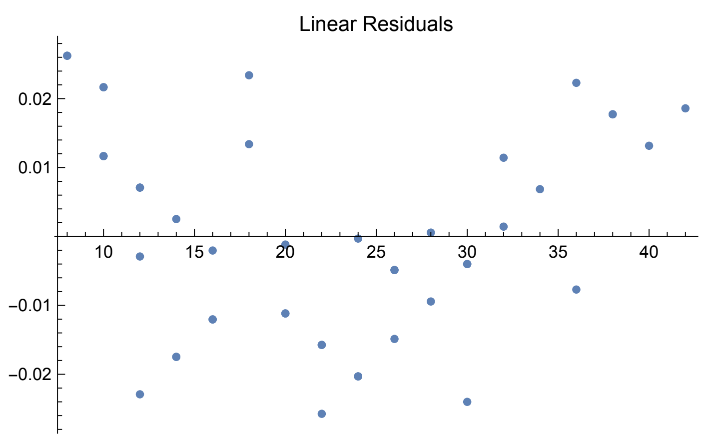
	DF	SS	MS	F-Statistic	P-Value
x	1	0.0295587	0.0295587	124.879	3.67471×10^{-14}
Error	42	0.00994133	0.000236698		
Total	43	0.0395			

Out[1302]= {{0.473622, 0.497398}, {−0.00320741, −0.00222616}}

In[1303]:=
```
p1 = ListPlot[data, PlotLabel → "Chlorine Fraction in a Unit of Product",
    AxesLabel → {"Weeks", "Fraction"}];
p2 = Plot[lm[x], {x, 0, 50}];
lmPlot = Show[p1, p2]
```

Out[1305]=


In[1306]:=
```
lrPlot =
 ListPlot[Transpose[{xs, lm["FitResiduals"]}], PlotLabel → "Linear Residuals"]
```

Out[1306]=


In[1307]:=
```
nlm = NonlinearModelFit[data, a + (0.49 - a) E^(-b (t - 8)), {a, b}, t]
nlm["ParameterTable"]
nlm["AdjustedRSquared"]
nlm["ANOVATable"]
nlm["ParameterConfidenceIntervals"]
```

Out[1307]= FittedModel[$0.39014 + 0.09986\, e^{-0.101633\,(-8+t)}$]

Out[1308]=

	Estimate	Standard Error	t-Statistic	P-Value
a	0.39014	0.00504494	77.333	6.34255×10^{-47}
b	0.101633	0.0133603	7.60709	1.99351×10^{-9}

Out[1309]= 0.999344

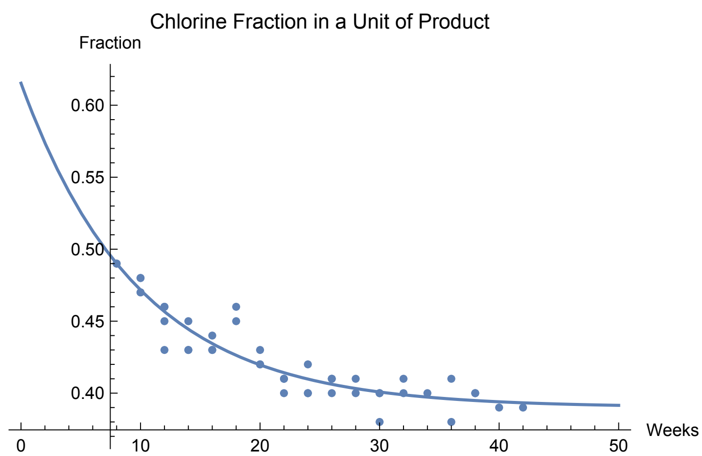
Out[1310]=

	DF	SS	MS
Model	2	7.982	3.991
Error	42	0.00500168	0.000119088
Uncorrected Total	44	7.987	
Corrected Total	43	0.0395	

Out[1311]= {{0.379959, 0.400321}, {0.0746706, 0.128595}}

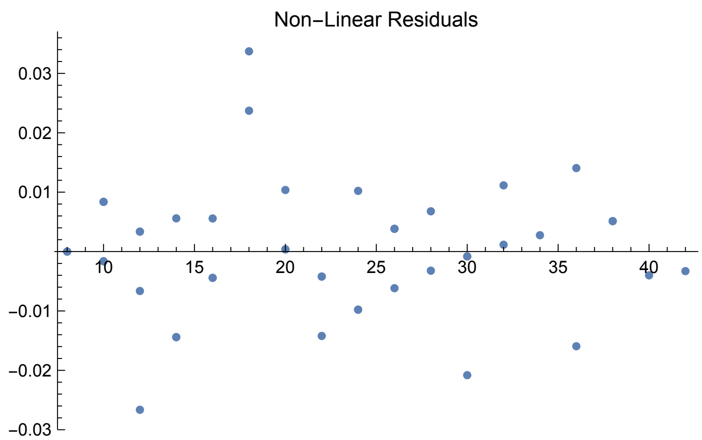
In[1312]:=
```
p2 = Plot[nlm[x], {x, 0, 50}];
nlmPlot = Show[p1, p2, PlotRange -> All]
```

Out[1313]=



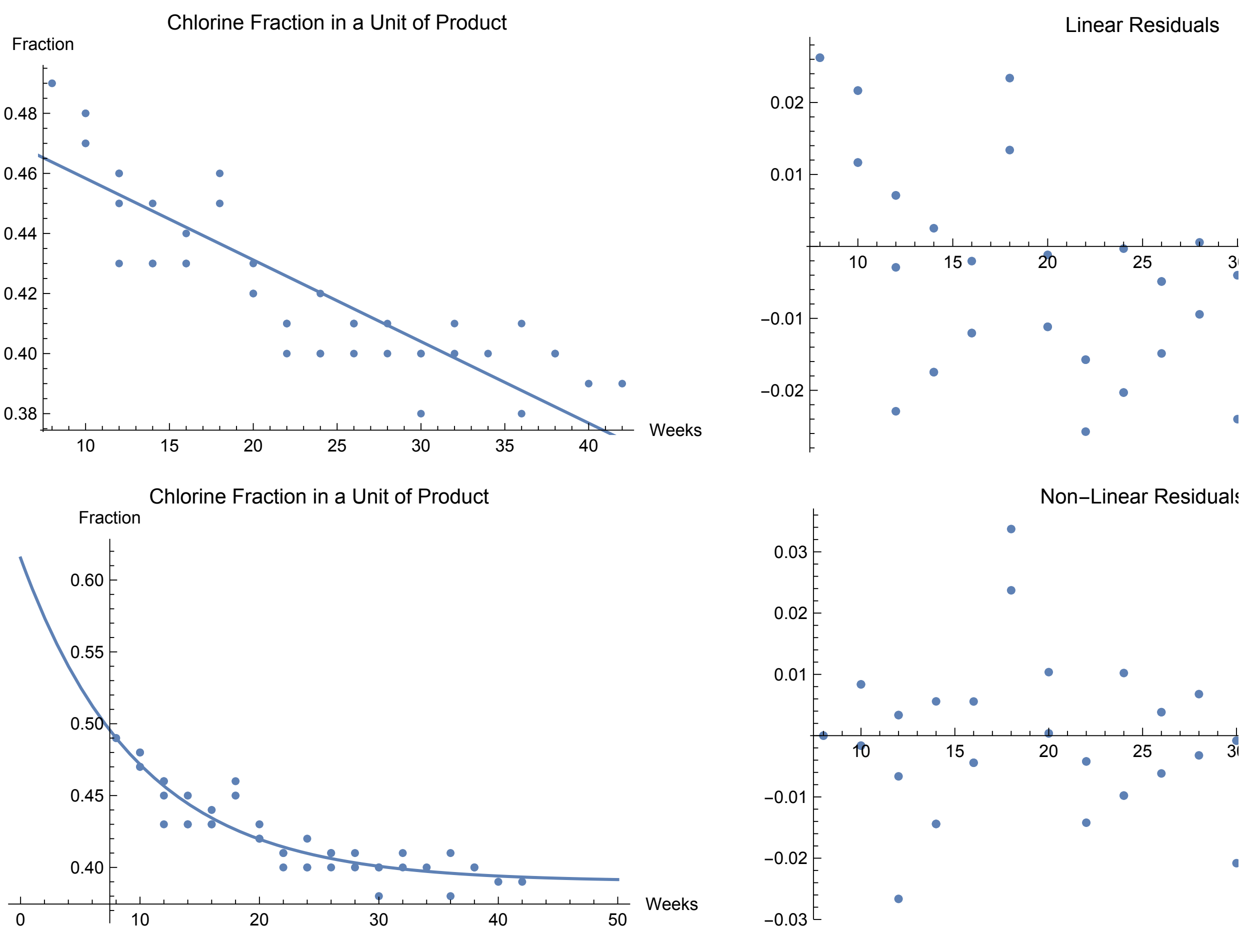
In[1314]:=
```
nlrPlot =
  ListPlot[Transpose[{xs, nlm["FitResiduals"]}], PlotLabel → "Non-Linear Residuals"]
```

Out[1314]=



In[1315]:=
```
GraphicsGrid[{{lmPlot, lrPlot}, {nlmPlot, nlrPlot}}]
```

Out[1315]=



The non-linear regression model is obviously preferable for a variety of reasons.

1. It has an asymptote. The linear model will eventually reach 0, and then go negative.
2. It has a better residual plot. There is a bit of a banana in that linear residual plot, whereas the residu-

als are pretty well the same across the data domain.
3. The average square residual is half that of the linear model.
4. We can’t compare those R^2 values. Anyone who uses those loses points. In class we discussed the fact that non-linear regression R^2 values aren’t accepted in some quarters.
5. We can’t compare sizes of confidence intervals -- we can note, however, that all parameters are not zero; that the two models use the same number of parameters; and that

#4:
a. What were Fletcher’s perception about whether climate change was occurring in the data or not?

He thought that there were only random irregular cycles in weather -- he didn’t think that there were anything systematically cycling or changing long term, insofar as he could see.

b. What are our (current) expectations for Diurnal Temperature Range, and why? What is DTR?

Based no work around the world, we expect DTR (the difference between maximum and minimum values for a day) to decrease. This is partly due to minima rising faster than maxima, due to higher night-time temperatures, and due to increased cloud coverage during the day.

c. We have two different sets of climate normals for BG. What are they (where did they come from)? What do we mean by “climate normals”?

Fletcher himself provided one set (normals by month for max and min); the other came from NOAA, climate normals (max and min) for Bowling Green for every day of the year.

d. Describe our initial expectations about what we would see in the Fletcher data, and what things we have discovered since that agree with or defy those expectations.

We expected that we would find more max maxima occurring in recent decades (so more years from recent decades), and more max minima;
we expected to find more min maxima and min minima in early decades.

We were also expecting to find a violation of a uniform distribution for the decades among records.

We did find a violation of the uniform, but it didn’t really accord with our expectations. We have since learned this change expected in the DTR, and are finding it in our data as well (the Custar data).

We also learned that we can’t really compare the early years with the later years, because of the introduction of thermometers that do a much better job of discerning the max for the day and the min for the day (two thermometers each doing one job).