Applied Math Modeling Exam 2 (Spring 2020)

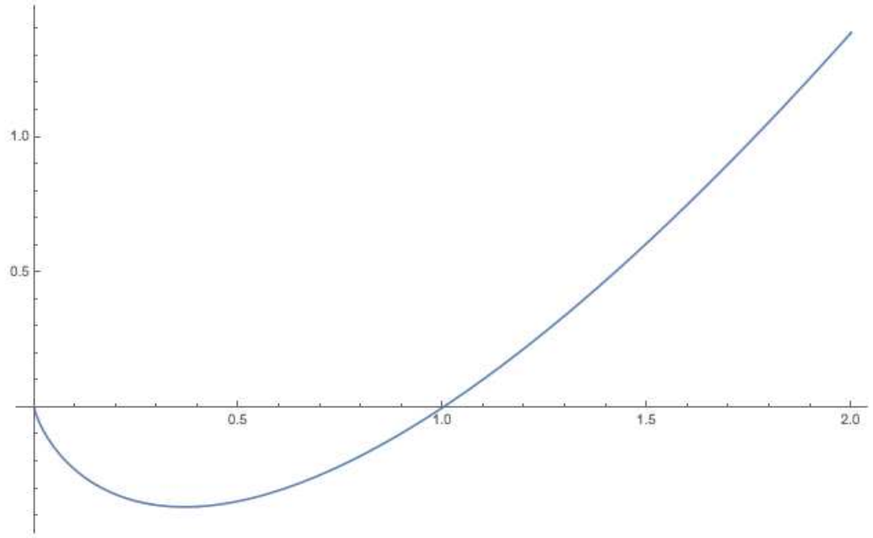
Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts) Illustrate Newton's method, using three iterations of Newton's method to approximate a root of the function

$$f(x) = x\ln(x)$$

starting from $x_0 = .5$. Include the calculation of each successive iterate.



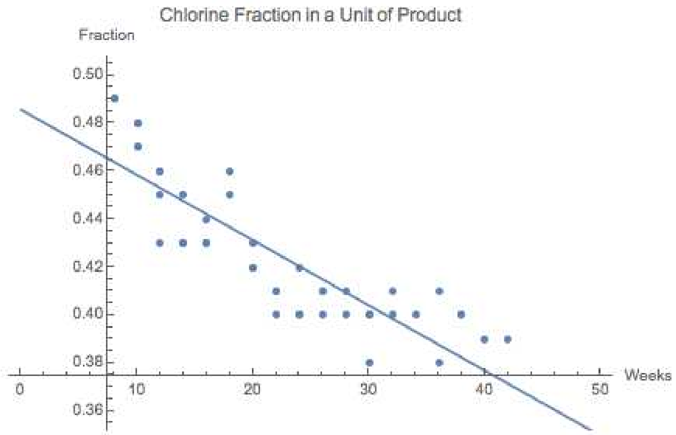
Problem 2. (10 pts) This is taken from an investigation performed at Procter and Gamble and reported in "Some reliability problems in the chemical industry", *Industrial Quality Control*, Vol21, 1964, #2, pp. 65-70. It involved a product A which must have a fraction 0.50 of Available Chlorine at the time of manufacture. The fraction of Available Chlorine in the product decreases with time. In the eight weeks before the product reaches the consumer a decline to a level of 0.49 occurs.... To assist management in decisions such as (1) When should warehouse material be scrapped? (2) When should store stocks be replaced?, etc. cartons of the product were analyzed over a period to provide the data shown here.

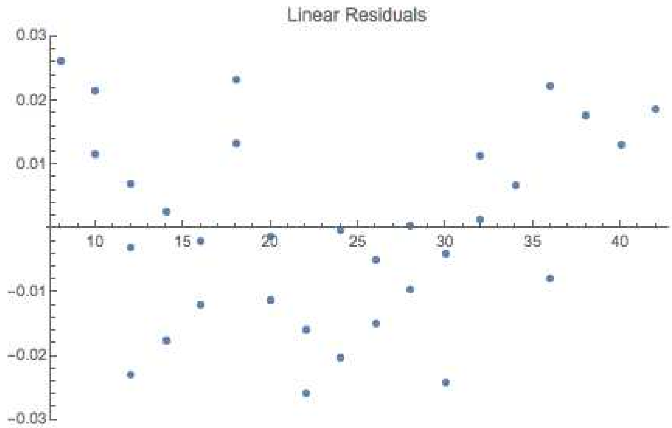
a. A nonlinear model of the form

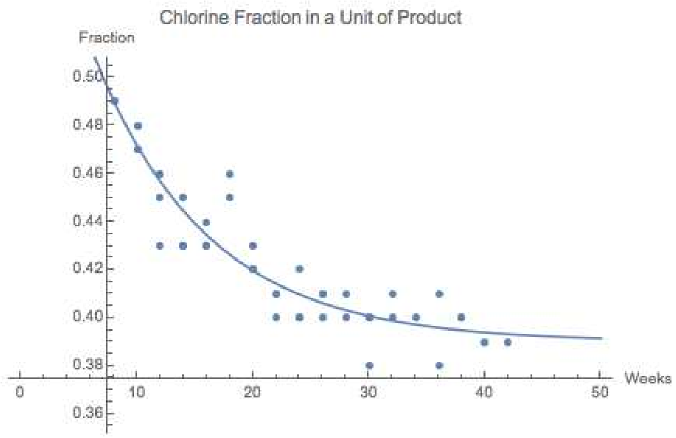
$$Y = \alpha + (0.49 - \alpha)e^{-\beta(X-8)}$$

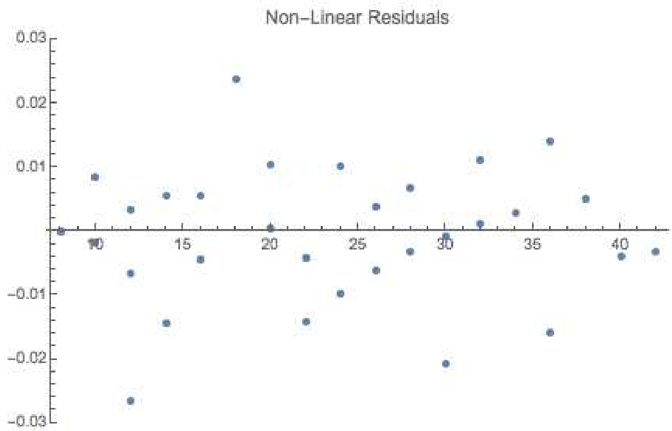
was proposed. Based on the description above, and the graphics below, explain the choice of this model.

b. A linear model was also considered: below we compare the linear and non-linear models:









Problem 2, cont.. (10 pts) Compare the two models – one linear, the other non-linear. Reference the plots from the preceding page, and the regression diagnostics below. Decide which is the better, with reasons. **Fill all the available space below with your comments.**

```
lm = LinearModelFit[data, x, x]
lm["ParameterTable"]
lm["AdjustedRSquared"]
lm["ANOVATable"]
lm["ParameterConfidenceIntervals"]
Mean[lm["FitResiduals"] ^ 2]
```

FittedModel[$0.48551 - 0.00271679\,x$]

	Estimate	Standard Error	t-Statistic	P-Value
1	0.48551	0.00589066	82.4204	4.44198×10^{-48}
x	−0.00271679	0.000243115	−11.1749	3.67471×10^{-14}

0.742328

	DF	SS	MS	F-Statistic	P-Value
x	1	0.0295587	0.0295587	124.879	3.67471×10^{-14}
Error	42	0.00994133	0.000236698		
Total	43	0.0395			

{{0.473622, 0.497398}, {-0.00320741, -0.00222616}}

0.000225939

```
nlm = NonlinearModelFit[data, a + (0.49 - a) E^(-b (t - 8)), {a, b}, t]
nlm["ParameterTable"]
nlm["AdjustedRSquared"]
nlm["ANOVATable"]
nlm["ParameterConfidenceIntervals"]
Mean[nlm["FitResiduals"] ^ 2]
```

FittedModel[$0.39014 + 0.09986\,e^{-0.101633\,(-8+t)}$]

	Estimate	Standard Error	t-Statistic	P-Value
a	0.39014	0.00504494	77.333	6.34255×10^{-47}
b	0.101633	0.0133603	7.60709	1.99351×10^{-9}

0.999344

	DF	SS	MS
Model	2	7.982	3.991
Error	42	0.00500168	0.000119088
Uncorrected Total	44	7.987	
Corrected Total	43	0.0395	

{{0.379959, 0.400321}, {0.0746706, 0.128595}}

0.000113675

Problem 4. (10 pts) Regarding the Fletcher project, answer the following:

a. What were Fletcher's perception about whether climate change was occurring in the data or not?

b. What are our (current) expectations for Diurnal Temperature Range, and why? What is DTR?

c. We have two different sets of climate normals for BG. What are they (where did they come from)? What do we mean by "climate normals"?

d. Describe our initial expectations about what we would see in the Fletcher data, and what things we have discovered since that agree with or defy those expectations.