

§9.3(a) #2, 11, 20, p 750-

(5) #25a, 34, 68, p 750-

a) #2 a) Find all input sequences yielding an output sequence of **001110**.

(This is called an "inverse problem".)

0 is the output of  $S_0$ ,

assuming that we've got the counting that

1st 0 no matter what. So we need to tell what six digit string will generate 011110 starting from  $S_0$ .

that produces a 1

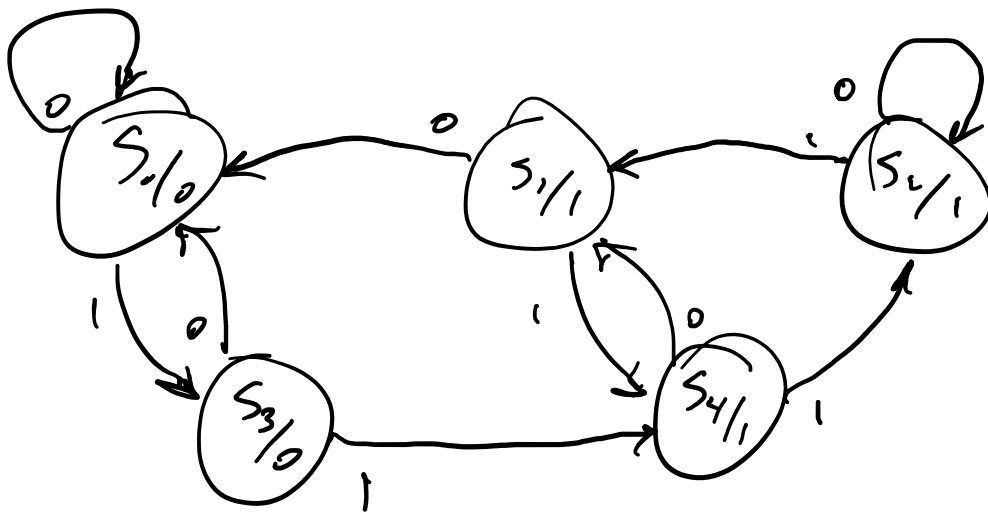
We have to get to a state  $s_n$  on a path of length 2 from  $S_0$ , & there's only one way to do it: 11. Now we're sitting on  $S_4$  & need to produce 110 - so we have to land on a state writing 0 in a path of length 3, writing 1s as we go. There's only one way

to do it:  $s_4 \xrightarrow{1} s_2 \xrightarrow{1} s_1 \xrightarrow{0} s_0$

So there's only one way to create that

output: **11110**  
(if we assume we get the 0 for free).

input	1	1	1	1	0	
state	$S_0$	$S_2$	$S_4$	$S_2$	$S_1$	$S_0$
output	0	1	1	1	1	0



If we don't count the 1<sup>st</sup> output of S

then: ① We need to get to 01 in a path of length 2, so 11. Write 0 S<sub>4</sub>

② Now we need to get to 0110, in a path of length 4.

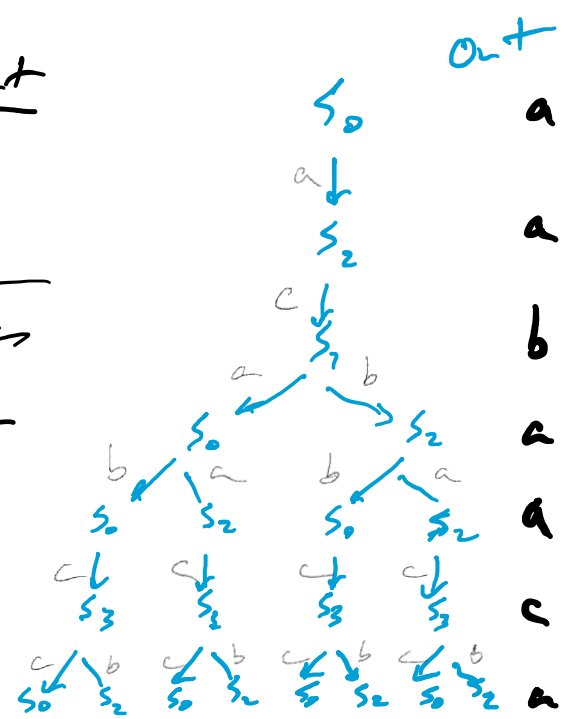
Two options: S<sub>1</sub> S<sub>4</sub> S<sub>1</sub> S<sub>0</sub> : 010  
 S<sub>2</sub> S<sub>2</sub> S<sub>1</sub> S<sub>0</sub> : 101

∴ 110100 or 111010

26.

	a	b	c	Out
S <sub>0</sub>	S <sub>2</sub>	S <sub>0</sub>	S <sub>3</sub>	a
S <sub>1</sub>	S <sub>0</sub>	S <sub>2</sub>	S <sub>3</sub>	b
S <sub>2</sub>	S <sub>2</sub>	S <sub>0</sub>	S <sub>1</sub>	a
S <sub>3</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>0</sub>	c

Produce abacaca



So a tree provides a good format for solving this problem.

$$a \in (arb)(arb) \in (crb)$$

2c. The machine's states produce 0s for all states but  $S_3$ ; so it's just a question of how we can end up in  $S_3$ .

$S_1, S_3^*$	1 1 1 1 1	1 1 1 1 0	1 0 1 1 1
$S_1, S_3, S_3^*$	0 1 1 1 1	1 1 1 0 0	1 0 1 1 0
$S_1, S_2, S_3, S_3^*$	0 0 1 1 1	1 1 0 0 0	1 0 1 0 0
$S_1, S_2, S_2^*, S_3^*$	0 0 0 1 1	1 0 0 0 0	0 1 0 1 1
	0 0 0 0 1		0 1 0 1 0
	0 0 0 0 0		

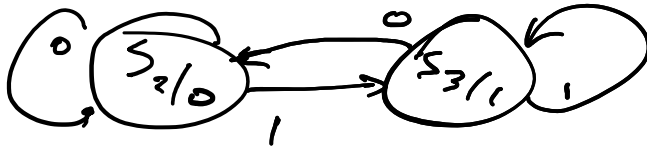
Looks like "what can't you get?"  
 Would be a better question... :)

You can get them all,  $2^5 = 32$  outputs. But more specifically,

input	$a_1, a_2, a_3, a_4, a_5$
state	$S_0$
output	0 $a_1, a_2, a_3, a_4, a_5$

Each state writes its input!

R is a delay FSM! And it could be minimized:

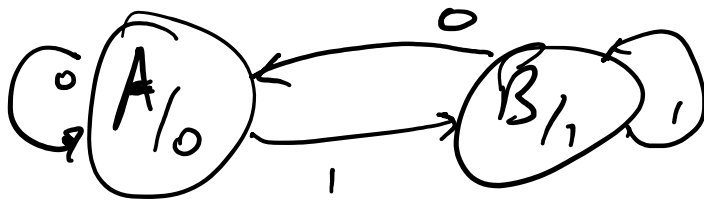


Let's see:

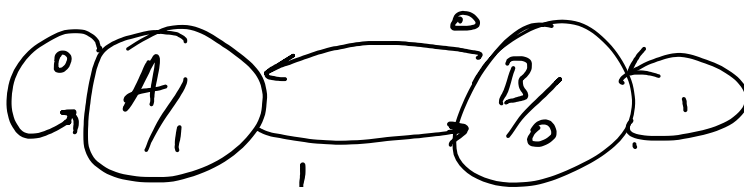
0-equivalent: {0, 1, 2}, {3}

1-equivalent: {0, 1, 2}, {3} Done!

It's that easy... A B →

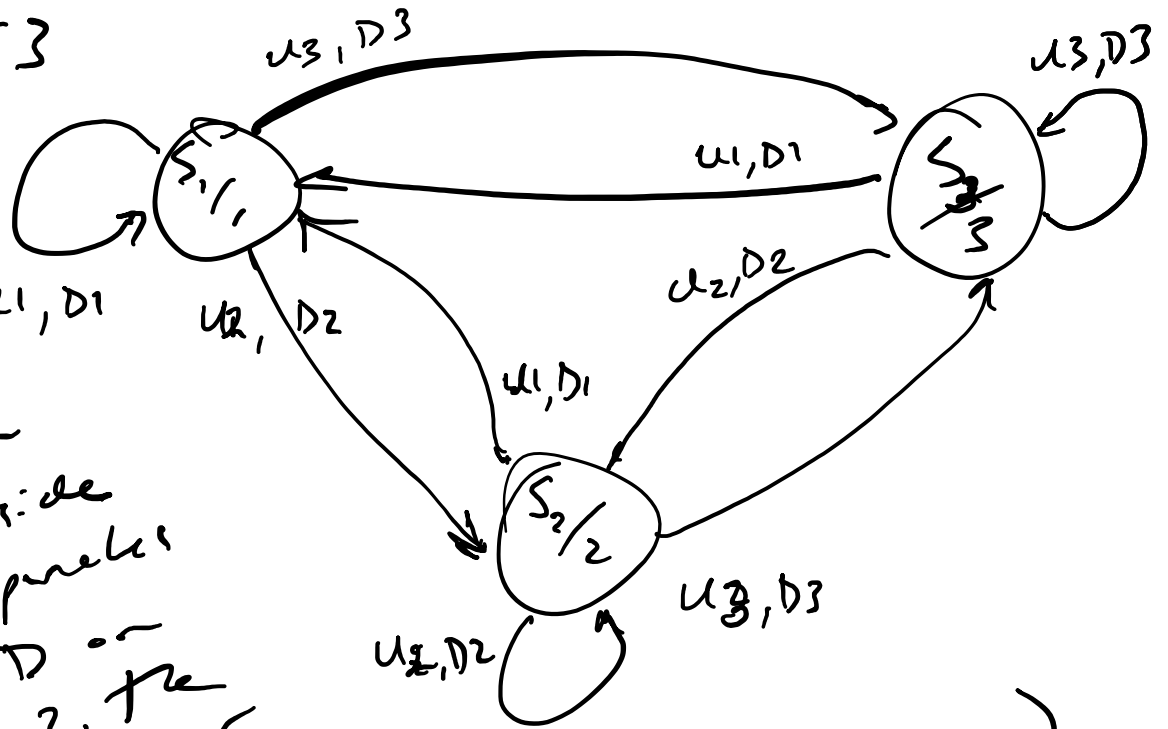


#11 a. Let's repurpose the machine above - which writes the input string, to write the opposite:



In	0	1	0	1	1
	A	A	B	A	B
	1	1	0	1	0

#20, 753



My interpretation was that when someone outside the elevator pushes either U or D on floor 2, the elevator

responds by going to that floor - or staying, if already there

Part 5) #25a, 34, 68

#25a : set of all strings containing an even # of 0s : (that includes 0 0s, so)



#34 There's a wastebasket state... if we fall into it, we're done.

00 is recognized, the 1<sup>st</sup> string.  
Then, so long as we keep getting 0s,  
the state is one of recognition.

1\* 00 0\*

We can pad w/ 1s  
at the outset, too.

#68 0-equivalent states:

{0, 1, 3, 6}, {2, 4, 5, 7}

1-equivalent:

{0, 3}, {1, 4}, {2, 4, 5, 7}

2-equivalent:

{0, 3}, {1, 4}, {2, 4}, {5, 7}

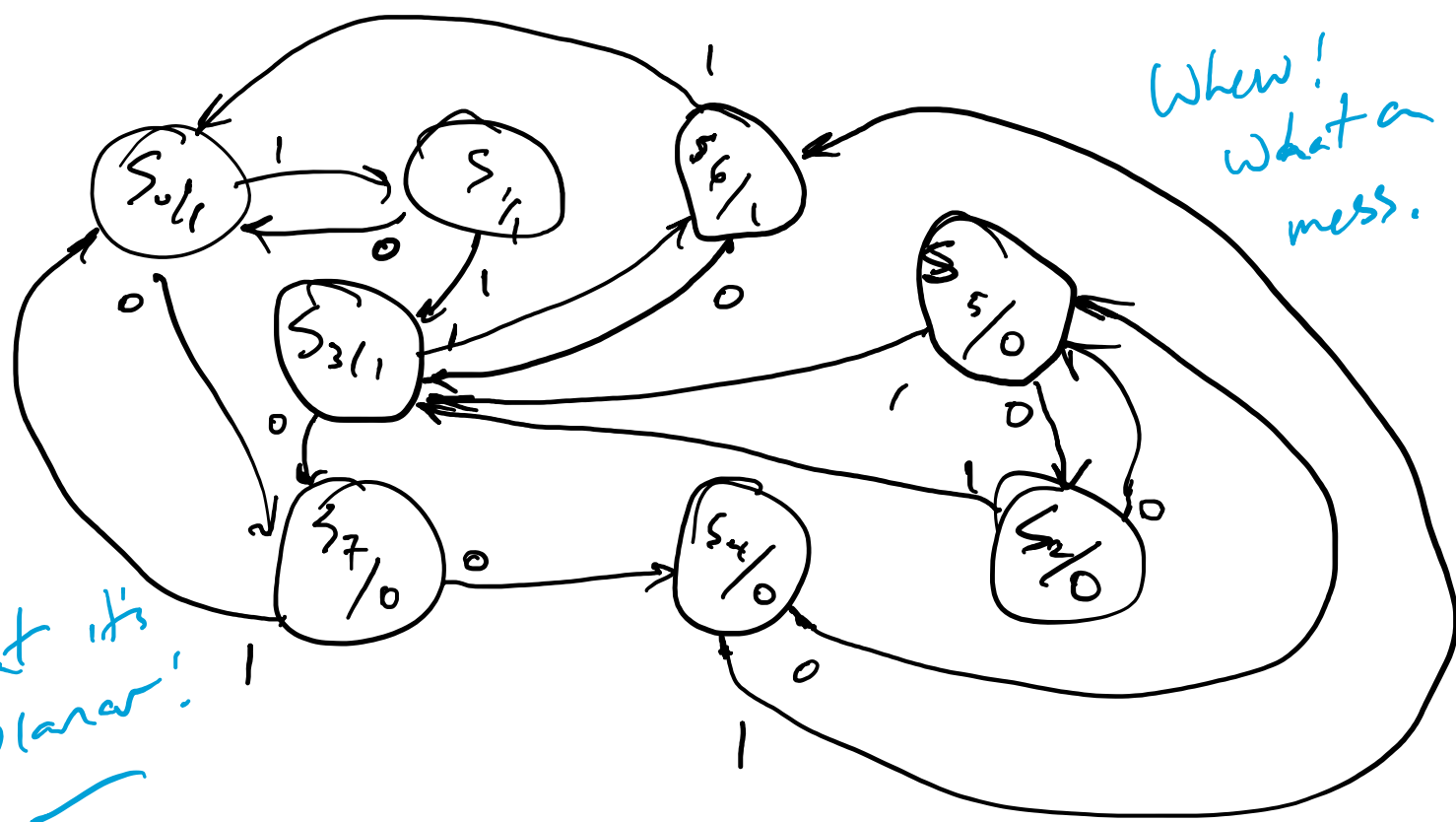
3-equivalent:

{0, 3}, {1, 4}, {2, 4}, {5, 7}

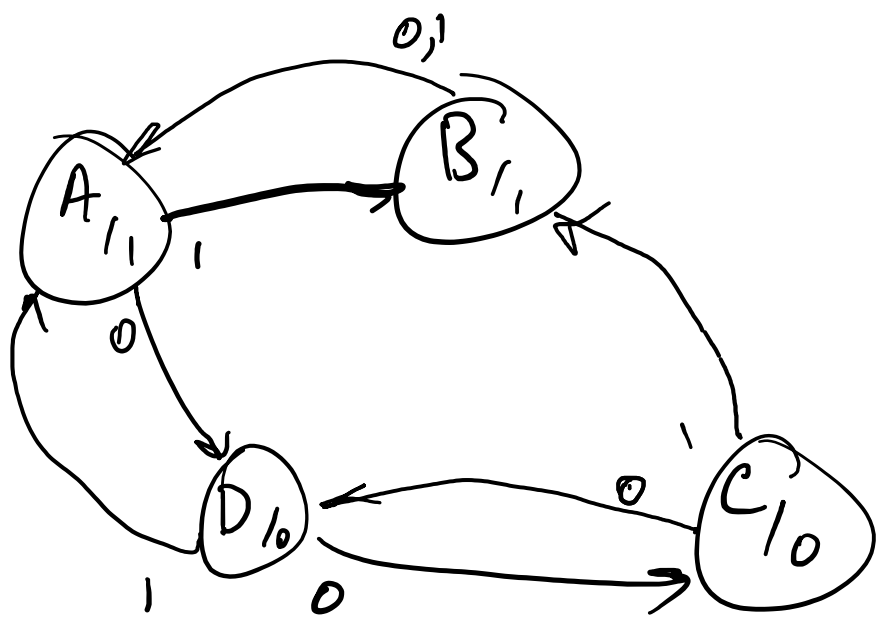
A      B      C      D

Done!  
No  
change.

When!  
What a  
mess.



But it's  
planar!



Simplified  
FSM,  
+ we  
can't  
simplify  
further:

A + B can't be equivalent.

$$\left. \begin{array}{l} A, 0 \rightarrow D/0 \\ B, 0 \rightarrow A/1 \end{array} \right\} + \left. \begin{array}{l} D, 1 \rightarrow A \\ C, 1 \rightarrow B \end{array} \right\} \text{ + these aren't equivalent.}$$