

MAT385 Test 2 (Spring, 2020): 4.1, 6.1, 6.2, 6.3

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (17 pts)

- a. (4 pts) Create a binary search tree by entering this line of Lewis Carroll's poem "Jabberwocky" in order: "long time the manxome foe he sought".

- b. (9 pts) Write the first lines of the new "poems" resulting from the following traversals:

- (1 pt) in-order:

- (1 pt) pre-order:

- (1 pt) post-order:

- i. (1 pt) What's curious and special about the in-order traversal "poem"?

- ii. (1 pt) What's the worst case number of comparisons for a binary tree search for a word not on this list? (And provide a word that would trigger it.)

iii. (2 pts) What's the **best** worst case number of comparisons, if we had entered the data in a different order? (How many nodes fit in a binary tree of depth d ?)

iv. (2 pts) Create a more "balanced" binary search tree by adding the words in a better order.

c. (4 pts) Draw two different trees that have

- inorder traversal r, a, c, e, c, a, r
- postorder traversal r, a, c, e, c, a, r
- preorder traversal r, a, c, e, c, a, r

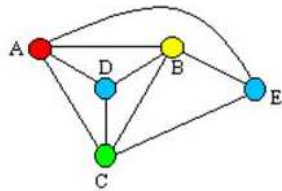
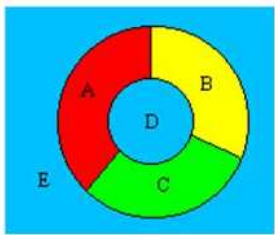
Problem 2: (12 pts) Consider the set of three colors, $S = \{R, G, B\}$ (red, green, blue), which we use to create any “RGB” color.

a. (4 pts) How many different colors can we create by combining (or not) each of the three (imagine adding some subset of these colors to white paint)? (Assume no gradations of color – either in or out – all or none.)

b. (2 pts) For a set S of n elements, what is the size of the powerset $P(S)$?

c. (2 pts) What can we say about the size (cardinality) of the powerset of an infinite set S ?

d. (4 pts) We certainly have enough colors to color any map, since we only need four:



Planar graph - a graph drawn in a plane without any of its edges crossing or intersecting

Each vertex (A,B,C,D,E) represents a region in a graph

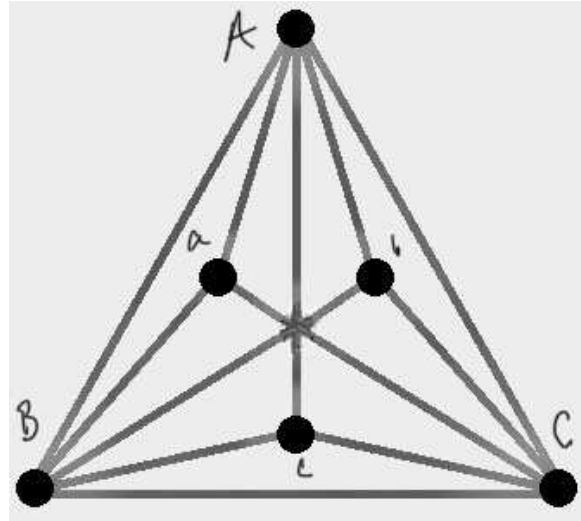
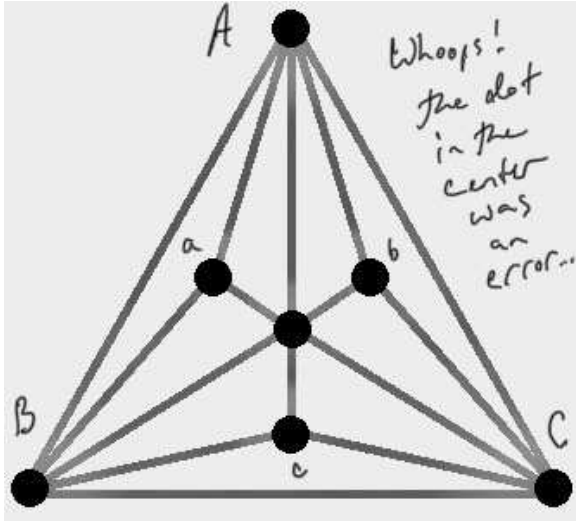
Each edge represents regions that share a boundary

i. (3 pts) The image above shows five countries requiring four colors, as well as the corresponding graph – nodes are countries, arcs are boundaries. It’s not the **simplest** map that requires four colors, however. Pare this map down to one still requiring four colors, and draw the corresponding graph.

ii. (1 pts) **Name** and describe the simplest graph that **requires** four colors.

Problem 4: (10 pts)

- a. (5 pts) The beautiful graph at left is clearly planar. At right is drawn another graph, where I have



eliminated the dot in the center – because it was a mistake. There wasn't supposed to be a node there – just false intersections. Demonstrate that the graph at right is **non-planar**. (Hint: it contains a famous non-planar graph as a sub-graph. You might redraw the graph to show this.)

- b. (5 pts) **Prove** (quite simply) that if we remove any one node from the graph at right, it IS planar. (Use **symmetry** – you don't need to make six arguments; or use a theorem about graphs with only five nodes.)

Academic Honesty Statement

At the moment, the university finds itself in unprecedented times. Faculty have worked to create exams that mimic the traditional face-to-face testing environment, and we are asking students to respond with honesty and integrity during this public health crisis. I have no doubt that you will respond in this way, but my role as your teacher also requires that I remind you of the Student Honor Code in the Code of Student Rights Responsibilities:

<https://inside.nku.edu/scra/information/students/rights-responsibilities.html>

The work you submit on this exam is bound by this policy. Below are some examples of behavior that would violate this code:

- Using unpermitted resources while completing the exam. This includes searching for answers or approaches on the Internet. It also includes asking (calling, texting, facetimeing, etc.) your classmates, friends, relatives, acquaintances, Siri, Alexa, or Google for assistance on the exam. Basically, if it's a behavior that your instructor would not allow while taking an exam during a traditional class meeting, you should also not do this during the online test.
- Submitting an exam completed by another person (either in whole or in part). Do not complete the exam and then share your responses with others. That would allow them to submit work that was not their own, and both individuals will be deemed in violation of the Honor Code.
- Discussing the exam with others in the course. When you submit your exam, you should not discuss the test with anyone else in the class until the due date has expired – for everyone. This includes texting and/or calling someone and asking if they've taken the test – that feels like an innocent question, but it can quickly become a conversation that violates the bullets above. During the exam window (basically throughout the day), you should only communicate with the instructor of the course, but no one else enrolled in the class.

Before submitting your exam, you will be asked if you have satisfied the Student Honor Code. Please respond with honesty and integrity.

When grading your work, if it is deemed you have violated the Student Honor Code, then all future exams must be completed with the use of Lockdown Browser with webcam monitoring. It will be your responsibility to ensure you have the proper technology to allow for this to occur. Additionally, the score earned on the exam in question will not be used when calculating your course average. I will communicate via email how this impacts the calculation of your grade in the course.

You are now be asked to sign the following statement, provided it is true:

The work submitted for this exam was completed in a way that did not violate NKU's Student Honor Code.

Signature:

Date: