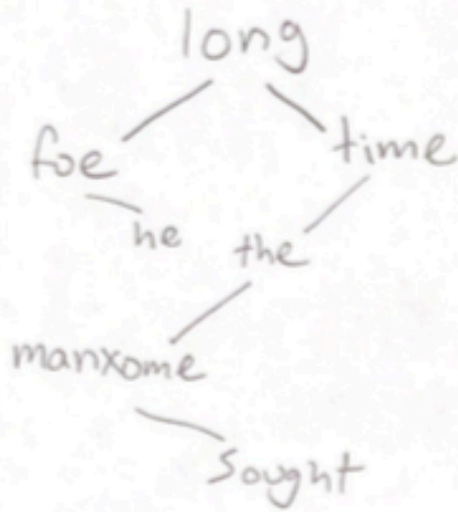
Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (17 pts) 16.5

a. (4 pts) Create a binary search tree by entering this line of Lewis Carroll's poem "Jabberwocky" in order: "long time the manxome foe he sought".

Nice work



b. (9 pts) Write the first lines of the new "poems" resulting from the following traversals:

- (1 pt) in-order: (L, root, R) foe, he, long, manxome, sought, the, time
- (1 pt) pre-order: (root, L, R) long, foe, he, time, the, manxome, sought
- (1 pt) post-order: (L, R, root) he, foe, long, sought, manxome, the, time

-.5

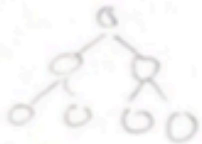
i. (1 pt) What's curious and special about the in-order traversal "poem"?

it is in alphabetical order

ii. (1 pt) What's the worst case number of comparisons for a binary tree search for a word not on this list? (And provide a word that would trigger it.)

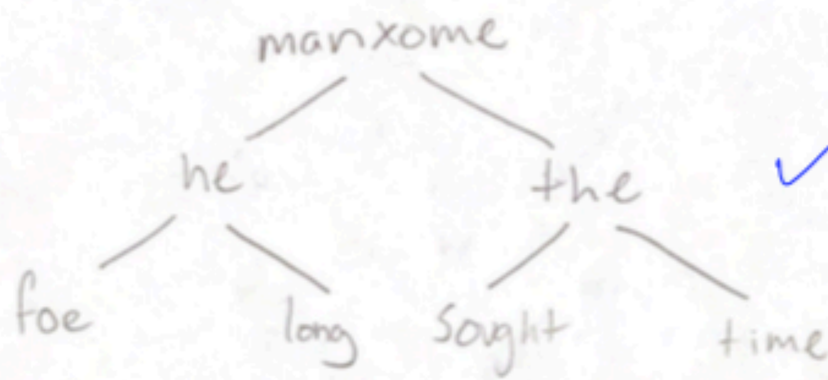
5, sour

iii. (2 pts) What's the **best** worst case number of comparisons, if we had entered the data in a different order? (How many nodes fit in a binary tree of depth d?)



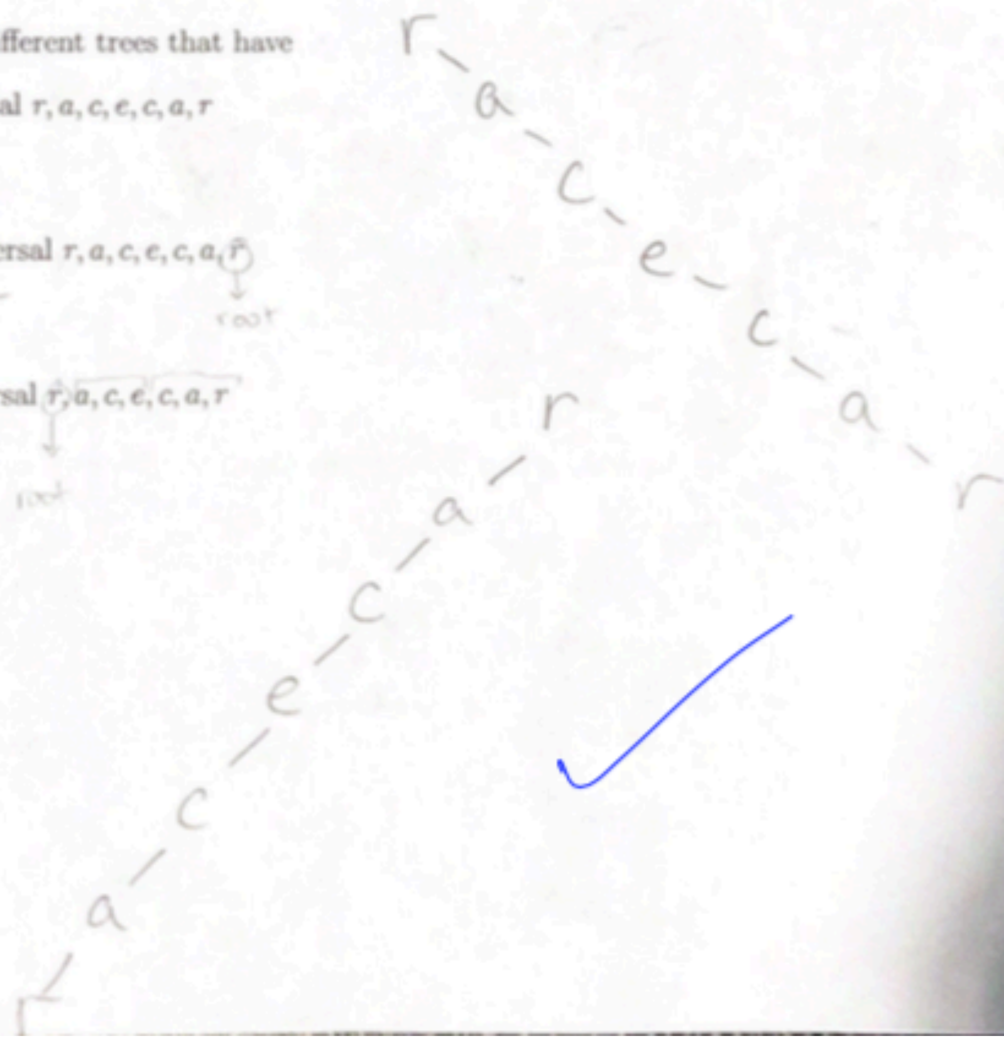
3 Comparisons

iv. (2 pts) Create a more "balanced" binary search tree by adding the words in a better order.



c. (4 pts) Draw two different trees that have

- inorder traversal r, a, c, e, c, a, r
  L, root, R
- postorder traversal r, a, c, e, c, a, r
  L, R, root
  root
- preorder traversal r, a, c, e, c, a, r
  root, L, R
  root



Problem 2: (12 pts) Consider the set of three colors, $S = \{R, G, B\}$ (red, green, blue), which we use to create any "RGB" color.

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a. (4 pts) How many different colors can we create by combining (or not) each of the three (imagine adding some subset of these colors to white paint)? (Assume no gradations of color – either in or out – all or none.)

$2^3 = 8$ combinations (or not)

b. (2 pts) For a set S of n elements, what is the size of the powerset $P(S)$?

2^n

c. (2 pts) What can we say about the size (cardinality) of the powerset of an infinite set S?

The cardinality of $P(S)$ will be larger than that of just S

good

d. (4 pts) We certainly have enough colors to color any map, since we only need four:

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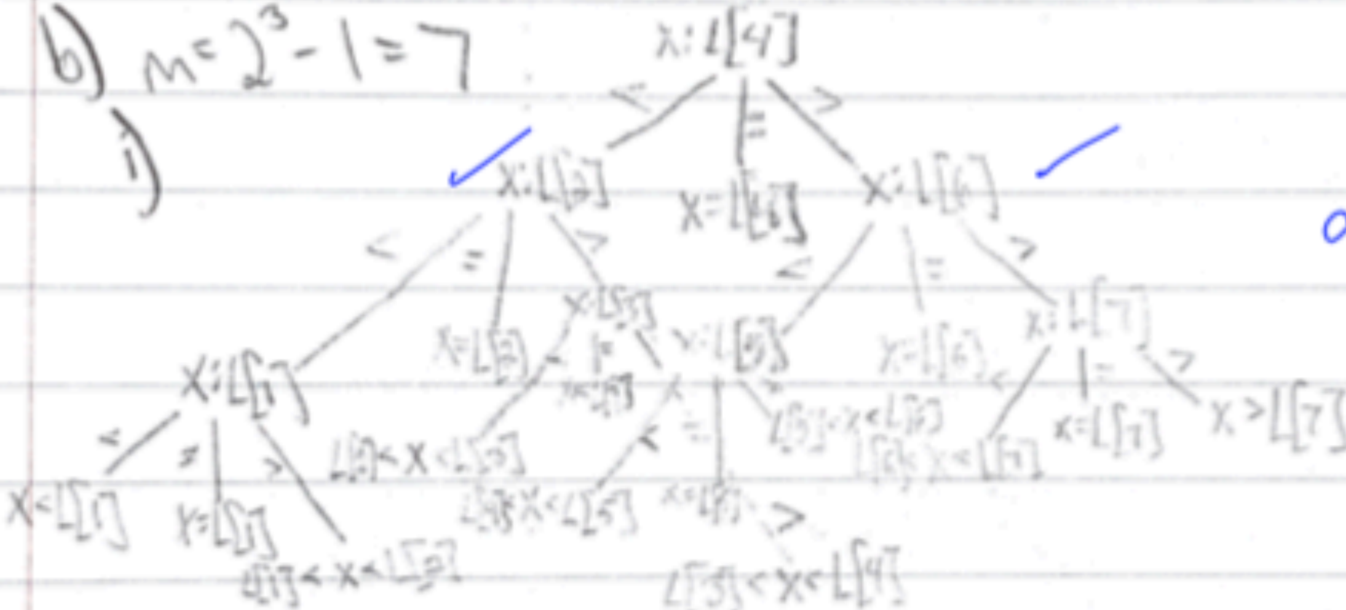
3. a) If the list length is odd, the 1st comparison is done at $\lceil \frac{n}{2} \rceil$ which would be the middle element for a binary search.

b) $n = 2^3 - 1 = 7$

i)

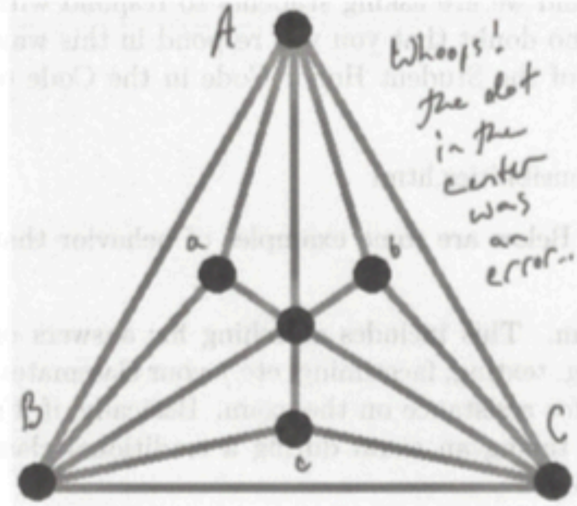


good

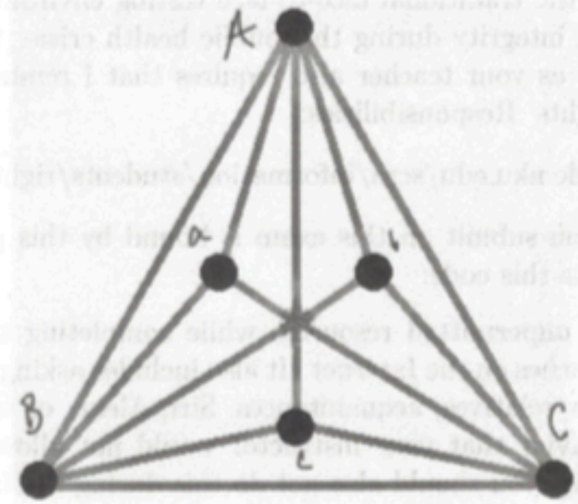
ii) $3+3+1+1+1+3+3 = \boxed{15 \text{ leaves/outcomes}}$

Problem 4: (10 pts) 10 Nice Work!

a. (5 pts) The beautiful graph at left is clearly planar. At right is drawn another graph, where I have

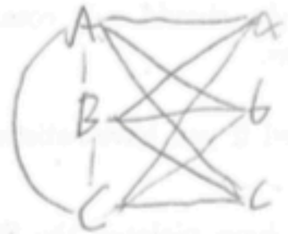


Whoops! the dot in the center was an error...



eliminated the dot in the center – because it was a mistake. There wasn't supposed to be a node there – just false intersections. Demonstrate that the graph at right is **non-planar**. (Hint: it contains a famous non-planar graph as a sub-graph. You might redraw the graph to show this.)

it contains $K_{3,3}$ ✓ as a subgraph.



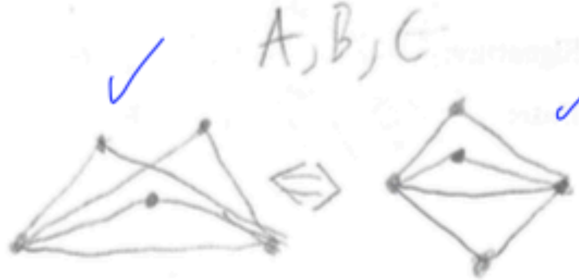
Good

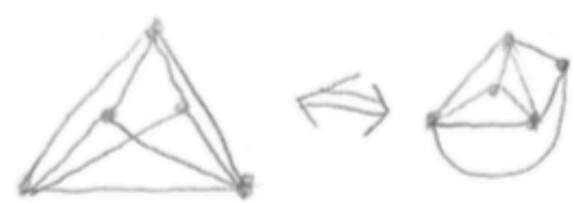
b. (5 pts) **Prove** (quite simply) that if we remove any one node from the graph at right, it IS planar. (Use **symmetry** – you don't need to make six arguments; or use a theorem about graphs with only five nodes.)

removing any node from the graph eliminates the $K_{3,3}$ thus allowing it to be planar

removing A, B, C

removing a, b, c





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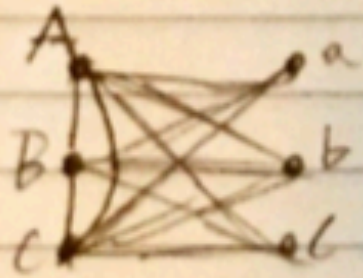
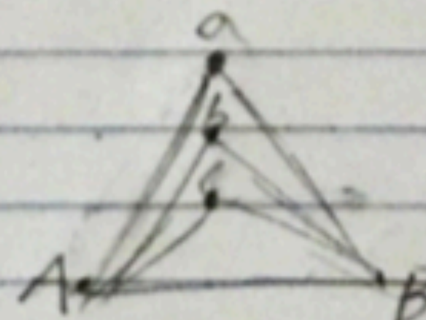
(4) a)



The graph contains $K_{3,3}$ which is non planar

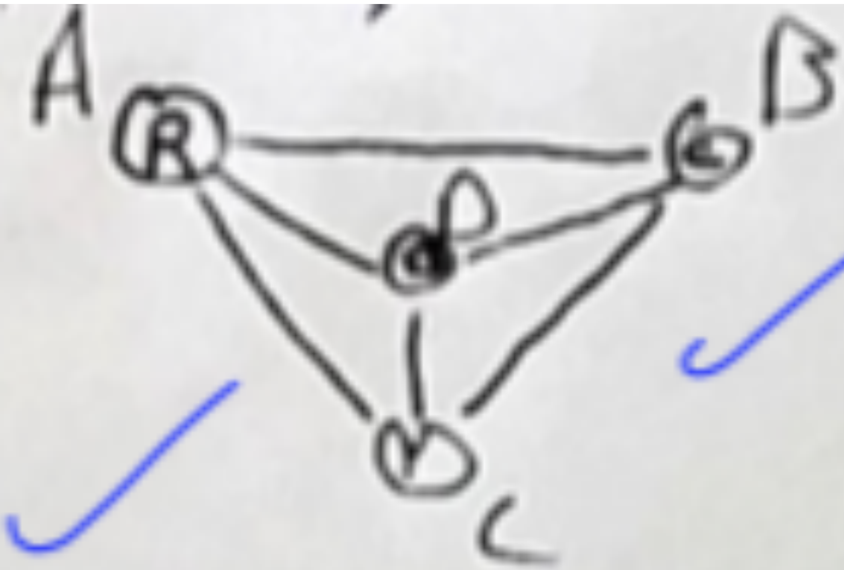
b) The graph on the right contains $K_{3,3}$ which by Kuratowski's theorem makes the graph non-planar. When a node is removed, the graph no longer contains either K_5 or $K_{3,3}$ which means it is planar

remove A



Nice!

d



K_4

Complete planar graph
connected

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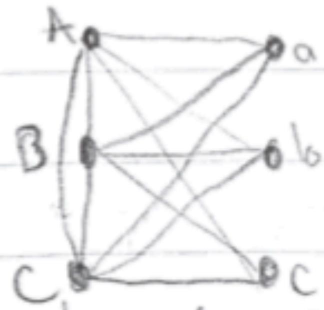
4, a) The graph has a subgraph that is $K_{3,3}$. So, There is no way that this graph can be planar.



Good

$K_{3,3}$ + K_5 are always not planar.

4. b) If we remove a node, like a, b, or c, Then the only non-planar graph with 5 nodes is K_5. But if we get rid of c, the graph would be planar because a and b are not adjacent. So, the graph is not K_5 and K_5 is the only non-planar graph so it has to be planar. Because if it is not fully connected, it is not K_5 and if it is not K_5, then it has to be planar.

Great!