Overview: Chapters 1–4

Chapter 1: Logic

Two binary $\wedge$ & $\vee$
One unary negation

Propositional: truth tables

Predicate: introduces variables

Useful in defining sets

in calculus $\forall x$ $\exists x$

Lewis Carroll's Game of Logic
Venn diagrams

Started proving things, based on some properties of $\wedge$ & $\vee$ –

commutativity
associativity
distributivity
complements (negation)
identities (T & F, 1 & 0)

Prove theorems & put them in our toolbox.

Chapter 2 : proofs & induction

Proof techniques

- direct proof
- by contradiction (Tautology test)
- exhaustion (Four-color problem)
- "not" proof - counterexample
- contraposition

$$P \to Q \iff Q' \to P'$$

(distinguish the contraposition from the converse).

- Induction

First glimpse of infinity (Natural numbers $\mathbb{N}$)

Two forms of induction:

1st principle -
2nd principle - } equivalent

Could be finite
Backwards induction
one couldn't be executed

All horses are the same color proof

Dominoes

an example of the temporary hypothesis.

1^{st} principle -

1. Base case ($P(1)$)
2. $P(n) \to P(n+1)$

Inductive step

if the n^{th} domino falls, then the $n+1^{th}$ domino falls

$\therefore P(n) \ \forall n \in \mathbb{N}$ (all dominoes fall)

2^{nd} principle -

1. Base case ($P(1)$)
2. $P(k) \ \forall k \,/\, 1 \le k \le n$

$\to P(n+1)$

All the dominoes have fallen up to n

The $n+1^{th}$ domino falls

$\therefore P(n) \ \forall n \,/\, n \in \mathbb{N}$.

Induction concerns over smallest infinity - a countable, denumerable infinity, $\mathbb{N}$.

Chapter 3 : Recursion

do it; then do it again, & again, n....
until satisfied.

Idea behind Newton's method
from calculus for solving
$f(x) = 0$

Defining objects (e.g. wffs).
We've defined trees recursively,
palindromic strings, & even
factorials :

base case

induction step

① $0! = 1$

② $P(n+1) = (n+1)P(n)$

Definition of $n!$ $\forall n \in \mathbb{Z}^+$

Recurrence relation
1st order, non-constant
homogeneous.

Another really important one:

Fibonacci numbers.

① $\begin{cases} F(1) = 1 \\ F(2) = 1 \end{cases}$ base cases

inductive step

② $F(n) = F(n-1) + F(n-2)$

Recursion - though a great way to define things - may be a horrible way to compute them!

Analysis of algorithms

- solve some in general
- proved the solution using induction.

(divide & conquer such as binary search).

Euclidean algorithm for finding GCD - Fibonacci #s were the worst case scenario.

Chapter 4: Sets

Three really cool ideas:

Venn Diagram

2^n

1. Power sets – the set of all subsets (powers of 2 – Pascal's triangle)

2. Infinity & beyond!

   There are infinitely many different sizes of infinity – all uncountable after the smallest infinity – $\aleph$.

   (Motel ∞; school buses).

3. Two binary $\cap$ + $\cup$ one unary complement

   same operations in propositional logic.

   → Boolean algebra