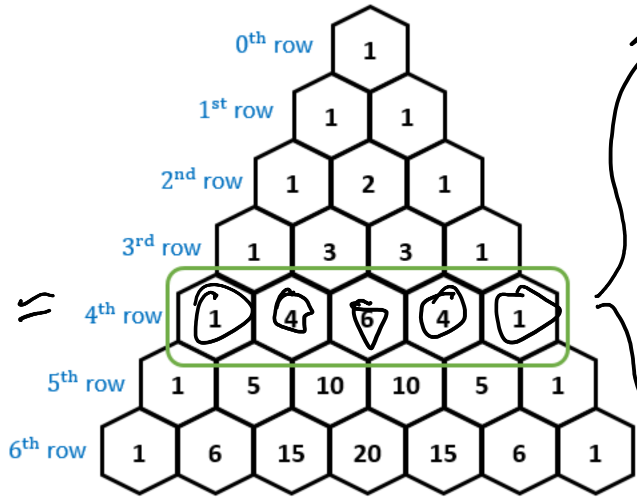
$16 = 2^4 =$



Set $S = \{A, B, C, D\}$

0 elements: $\{\ \}$ Empty set

1 element:

$\boxed{\{A\}}$, $\{B\}$, $\{C\}$, $\{D\}$,

2 elements:

$\{A, B\}$, $\{A, C\}$, $\boxed{\{A, D\}}$

$\{B, C\}$, $\{B, D\}$, $\{C, D\}$

3 elements:

$\{B, C, D\}$, $\{A, C, D\}$, $\{A, B, D\}$, $\{A, B, C\}$

4 element: $\{A, B, C, D\}$

The <u>Power Set</u> of S is the set of all subsets of S.

		0
1	$\leftrightarrow$ 0	1
2	$\leftrightarrow$ 1	2
3	$\leftrightarrow$ 2	3
4	$\leftrightarrow$ 3	4
5	$\leftrightarrow$ 4	5
6	$\leftrightarrow$ 5	6
7	$\leftrightarrow$ 6	7
	$\cdots$ 7	
$\mathbb{N}$		Whole numbers

The sets (right & left) are equal ~~#~~ in size, because they can be put into one-to-one correspondence.

$\mathbb{N}$		Even $\mathbb{N}$
1	$\longleftrightarrow$	2
2	$\longleftrightarrow$	4
3	$\longleftrightarrow$	6
4	$\longleftrightarrow$	8
5	$\longleftrightarrow$	10
6	$\longleftrightarrow$	12
$\vdots$		$\vdots$

Because these two sets can be put into one-to-one correspondence, they're the same size!

A proper subset of an infinite set may be exactly the same size as the set itself!

Bus 1 : 1, 2, 3, 4, ...

Bus 2 : 1, 2, 3, 4, ...

Bus 3 : 1, 2, 3, 4, ...

Bus 4 : 1, 2, 3, 4, ...

$\vdots \qquad \vdots$

How do we fill the Hotel?

Bus 1 : $n \to 2^n$

Bus 2 : $n \to 3^n$

Bus 3 : $n \to 5^n$

Bus 4 : $n \to 7^n$

$\vdots \qquad \vdots$

Bus m : $n \to (m^{th} \text{ prime})^n$

$\vdots \qquad \vdots$

In order for two individuals from different buses to end up in the same room, e.g. $3^r = 7^s$;

whatever room that is would have two different prime factorizations:

$$\underbrace{3^r = 7^s} \quad \text{where } \underline{r \,\&\, s \in \mathbb{N}}.$$

Two distinctly different prime factorizations.

Impossible!

Since we have an infinite # of primes, every bus gets their students into the Hilbert Hotel.