Graph Homework

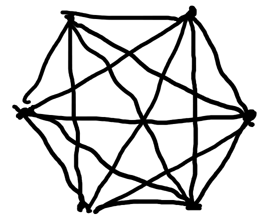
$\frac{(n-1)\,n}{2}$

1. Draw the complete graphs with 6, 7, and 8 vertices. How many edges are there for each? What is the formula for the number of edges of a complete graph with *n* vertices?
2. Draw all the distinctly different simple graphs with one and two vertices (there aren't many!). How many were there for three, four, and five vertices (these were done in the "lecture", or were in the notes)? Can you find any pattern to the number of each? How many do you think there are with six vertices?
3. A floor plan for a house can be considered a graph: each room is a vertex, and each door between rooms is a "bridge" (an edge).
    1. Create a "floor plan" of your house ([here's mine]). Does it have an Euler path? Explain why it does or doesn't. (By the way, the "outdoors" is also a vertex! Your front/back door(s) leads to this "outdoors" region. In an apartment, out into the hall.)

       You can design your own floor plan, if you wish -- perhaps of your dream house. I'm not going to be checking that your floor plan is a faithful representation!

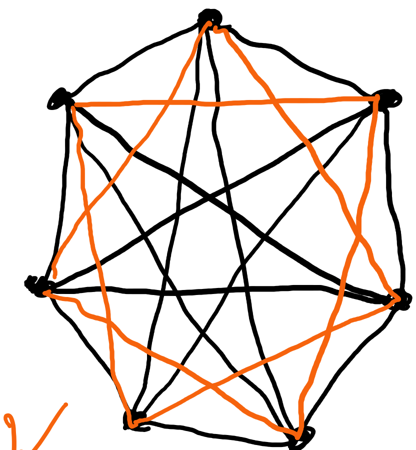
       If your house has multiple floors, pick your favorite.
    2. Add windows as additional edges (generally to the outdoors). Does that change things?
4. Give two examples of balanced and two examples of unbalanced graphs with four people in them (see "The Enemy of my Enemy is my Friend"). Four examples total.

#1



K_6

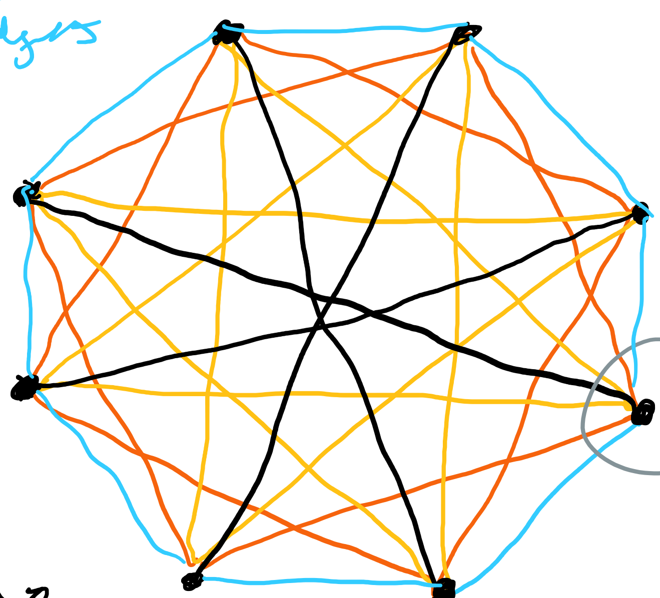
$\frac{5\cdot 6}{2} = 15$ edges



K_7

$\frac{6\cdot 7}{2} = 21$ edges

$\frac{7\cdot 8}{2} = 28$ edges



K_8

Each vertex of K_n of degree $n-1$

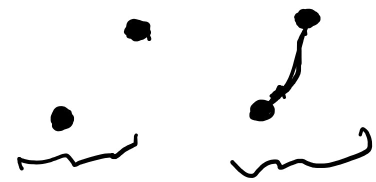
degree 7

#2. One vertex

K_1 [It's complete!]

(that's it! :))

Two vertices



isolated complete K_2

"duals" of each other.

Three vertices: 4

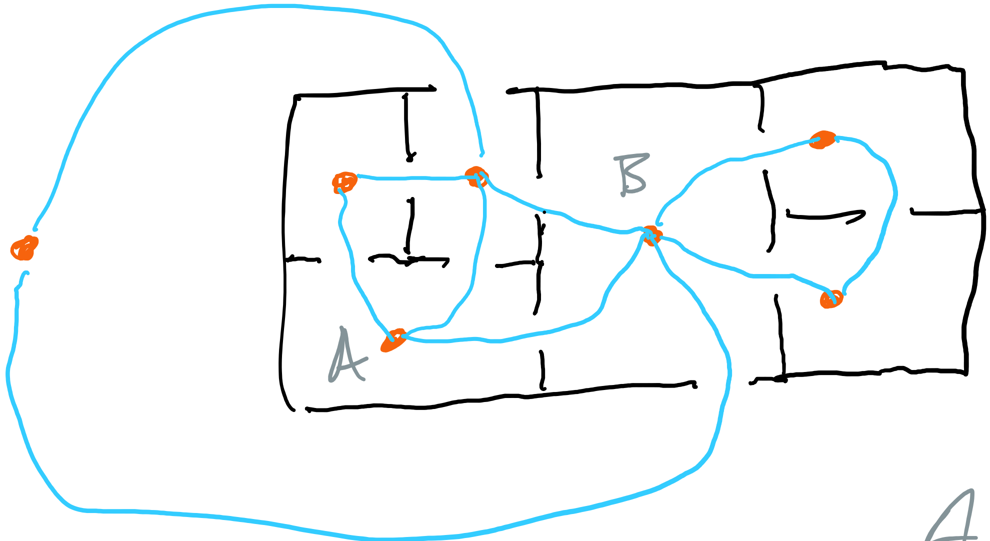
Four " : 11

Five " : 34

No clear Pattern.

Turns out there are 156 with six vertices. Wow!

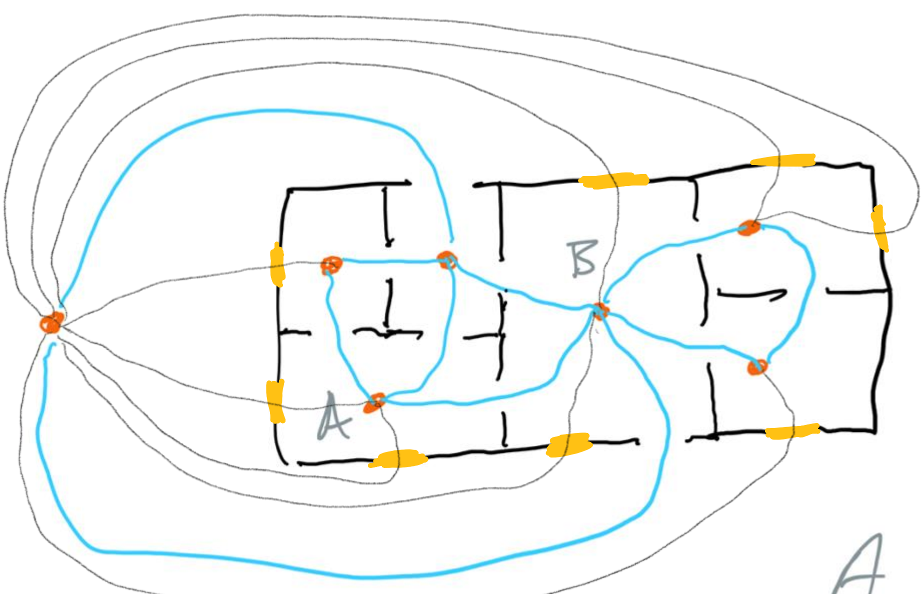
#3 - This will depend on your house plan. I'll just make one up here (I already posted my own on-line):



There's an Euler path — you'd start at A & end at B (or vice versa)

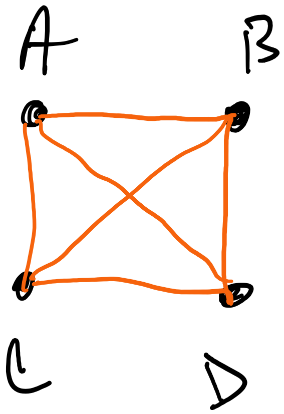
If you punch a bunch of windows you'll open "bridges" to the outside node.

Only two odd vertices, so an Euler path exists.

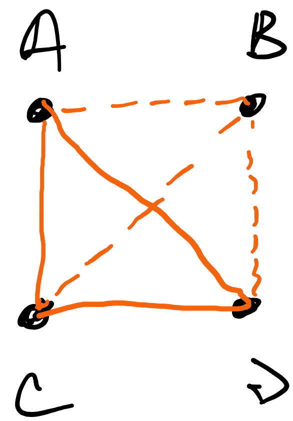


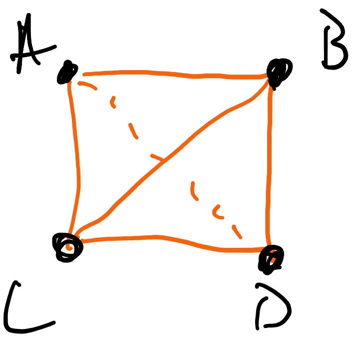
Now there are two many odd nodes. No more Euler path!

#4 Friends A, B, C, D:

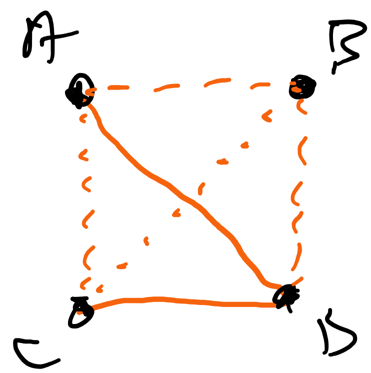


balanced





unbalanced



The answer isn't a unique answer. There are many different answers.