Day 13, MAT115

Last Time   Next Time

- Announcements:
  - I've made up a key out of your classmates work (hopefully your work, too!).
  - Materials for today:
    1. Here is a video covering today's topic of Egyptian division.
- Last time we had a look at
  1. the Fibonaccis, embedded in Pascal's Triangle (Fibonaccis lurk within Pascal's triangle).
  2. We also examined how we can put Pascal's triangle to use, in counting the number of possible Facebooks..
  3. Finally, we jumped into Egyptian multiplication, which relied on two things: the Egyptian's ability to double, and the **unique** factorization of a counting number as a sum of powers of 2.
- Today we're going to go beyond multiplication, and see how the Egyptians did division! Well, fractions, at any rate. And I know how you all love fractions.
  - We started last time with some simple examples, and we should do a warmup: what's 223*67?

$67 = 64 + 2 + 1$

$223 * 67 = 223 * (64 + 2 + 1)$

1	223	*
2	446	*
4	892	
8	1784	
16	3568	
32	7136	
64	14272	*
128	Too big!	

14272 + 446 + 223 = 14941

On the left we build the "missing part" of the product -- 67 -- and on the right we build up our answer, using the corresponding doubles of 223.

Remember that they would frequently start with the larger, to reduce the number of doublings.

  - Now: multiplication's not too bad. How about Egyptian division?
    - I've given you a reading to do for homework:
      - Take a look at your reading, p. 13: you'll see how Egyptians wrote their numbers. (Notice the blocking; also that they're base 10 people.)
      - Then, at the end of your reading, how they wrote fractions.
    - We can think of division as just using the multiplication table "backwards". So if we write the quotient (which is what we're looking for) as

dividend/divisor = quotient

We can think of this as a product instead:

divisor*quotient = dividend

For the product we'd take one of the parts of the product (the divisor, say), and double it on the right. Making up the quotient with numbers on the left, we'd then find the dividend by adding up the corresponding numbers on the right.

In the division problem we know the **dividend**, so we reverse the process: we find numbers on the right that sum to the dividend, and then add up the corresponding numbers on the left to give the quotient, which is what we're after.

Example: Let's try this one from last time (we'll just reverse the problem, to show how we're using the table backwards).

$23 * 42 = 966$

$966/23 = 42$

$32 + 8 + 2 = 42$

1	23	
2	46	*
4	92	
8	184	*
16	368	
32	736	*
64	Too big!	

Build a 966! 736 + 184 + 46 = 966

- Let's look at the simplest example imaginable: divide 32 by 8. We can actually do it by Egyptian multiplication, since 8 divides into 32 evenly:

$\frac{32}{8} = ?$ $? = 4$

1	8	
2	16	
4	32	*

So the answer is 4 (how do we get 4?)

- Similarly we could divide 40 by 8, using the same table (again easy, since 8 divides into 40 evenly):

$\frac{40}{8} = ?$ $? = 4 + 1 = 5$

1	8	*
2	16	
4	32	*

$32 + 8 = 40$

So the answer is 5 (how do we get 5?)

- Now: what happens when the division doesn't work out quite so nicely? We get "the f-word": fractions!
  - When the denominator doesn't divide the numerator evenly, fractions make it more *interesting* (my word -- you might use a different word!:).

    Let's look at an example: divide 35 by 8.

    In a way we turn it into a multiplication problem: what times 8 equals 35? So we know the 8, and use it to "double" -- but then to "halve", when 8 won't go evenly into 35:

$\frac{35}{8} = ?$ $? = 4 + \frac{1}{4} + \frac{1}{8}$

1	8	
2	16	
4	32	*
1/2	4	
1/4	2	*
1/8	1	*

$35 = 32 + 3 = 32 + 2 + 1$

So the answer is 4+1/4+1/8

  - the Egyptians restricted themselves to the so-called "unit fractions", which are fractions of the form *1/m*: unit fraction table, which is found on the Rhind Papyrus (which dates to around 1650 BCE).

    But they didn't restrict themselves to "halving", as our next example shows. Divide 6 by 7:

$\frac{6}{7} = ?$ $? = \frac{1}{2} + \frac{1}{4} + \frac{1}{14} + \frac{1}{28}$

1	7	
1/2	3+1/2	*
1/4	1+1/2+1/4	*
1/7	1	
1/14	1/2	*
1/28	1/4	*

$6 = 3 + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{2}$

So the answer is 1/2+1/4+1/14+1/28 (we usually order them from largest to smallest).

Notice that the Egyptians didn't use decimals -- you shouldn't either!

Why did Egyptians do things this way? (an example division problem, 3/5)

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   Dominic Olivastro, "Ancient Puzzles", suggests a third reason why
this use of unary fractions is good. Consider the problem Ahmes poses
of dividing 3 loaves of bread between 5 people. We would answer "each
person gets 3/5-ths of a loaf". If we implemented our solution, we might
then cut 2 loaves into 3/5 | 2/5 pieces, with bread for 3 people; then
cut one of the smaller pieces in half, giving the other two people
2/5 + 1/5 pieces. Mathematically acceptable, but try this with kids and
they will insist that it is not an even division. Some have larger pieces,
some have smaller.
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Ahmes would calculate 3/5 as :
   3/5 = ()3 + ()5 + ()15    [ = 1/3 + 1/5 + 1/15 ]
Now cut one loaf into fifths, cut two more into thirds, then take one of
the 1/3-rd pieces and cut it into 5-ths (for the 1/15-th pieces), and you
can now distribute everyone's 3/5-ths share in a way that _looks_ equal,
since they will have exactly the same size pieces. (And no, I don't want
to argue about the crust.)
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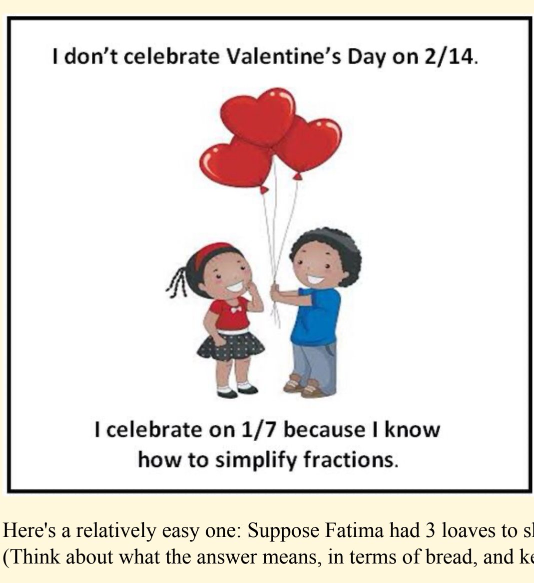
  - There is another way to get these answers, using the Fraudini trick and the Unit Fraction Table. So let's try those same examples but using the unit fraction table rather than our doubling tables.
    1. $\frac{35}{8} = \frac{32+2+1}{8} = 4 + \frac{1}{4} + \frac{1}{8}$ $= \frac{32}{8} + \frac{2}{8} + \frac{1}{8}$
    2. $$\frac{6}{7} = \frac{4+2}{7} = \frac{4}{7} + \frac{2}{7} = 2\frac{2}{7} + \frac{2}{7}$$

       so we look up $\frac{2}{7}$, and find that $\frac{2}{7} = \frac{1}{4} + \frac{1}{28}$. Therefore,

       $$\frac{6}{7} = 2 * (\frac{1}{4} + \frac{1}{28}) + \frac{1}{4} + \frac{1}{28} = \frac{1}{2} + \frac{1}{14} + \frac{1}{4} + \frac{1}{28}$$

    which is exactly what we got before -- except they're not in order from largest to smallest.



  - Here's a relatively easy one: Suppose Fatima had 3 loaves to share between 4 people. How would she do it? (Think about what the answer means, in terms of bread, and keeping kids happy.)
  - A little trickier:
    1. How would you divide 5 by 7?

       (Start with halves, and then what?)
    2. How can we use the unit fraction table to get the same answer?
  - How would you like to do story problems like this one?!

    Notice the numerals and fractional notation, as we see in our reading on page 13 and at the end.

- **Links**:
  - Unit Fraction Table

Website maintained by Andy Long. Comments appreciated.
Updated on 02/27/2021 22:30:19

$\frac{5}{7} = ?$ $? = \frac{1}{2} + \frac{1}{7} + \frac{1}{14}$

1	7	
1/2	3+1/2	*
1/7	1	*
1/14	1/2	*

$5 = 3 + \frac{1}{2} + 1 + \frac{1}{2}$

$$\frac{5}{7} = \frac{4+1}{7} = \frac{4}{7} + \frac{1}{7}$$
$$= 2\left(\frac{2}{7}\right) + \frac{1}{7}$$
$$= 2\left(\frac{1}{4} + \frac{1}{28}\right) + \frac{1}{7}$$
$$= \frac{1}{2} + \frac{1}{14} + \frac{1}{7}$$

14941