Day 15, MAT115

Last Time Next Time

- Announcements:
  - Your Egyptian Multiplication (and Facebooks) homeworks have been graded (key here).

    I thought that I'd use this homework as something of a motivation for why symmetry is important and useful, so let's look at that Facebook problem, and see how to use symmetry to help organize our Facebook world....
  - Your Egyptian division homework is due tonight. One thing that spooked one of your classmate was that **there is no unique way** to represent a given fraction as a sum of unit fractions!

    So, for example,

    $$\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$$

    and it turns out that, since

    $$\frac{1}{a}=\frac{1}{a+1}+\frac{1}{a(a+1)}$$

    for any positive integer a, we can replace the $\frac{1}{6}$ above as

    $$\frac{1}{2}=\frac{1}{3}+\left(\frac{1}{7}+\frac{1}{6*7}\right)=\frac{1}{3}+\frac{1}{7}+\frac{1}{42}$$

    and (do it again, do it again) you have an infinite number of ways of writing any fraction as a sum of unit fractions!

    This was no doubt a source of consternation for the Egyptians. We don't know exactly how they came up with their representations in the Rhid papyrus, but we speculate that they tried to find the representation with **the fewest number of terms**.

    Important: You can always check that you're right when you do an Egyptian division by simply computing both sides and seeing if they're equal!

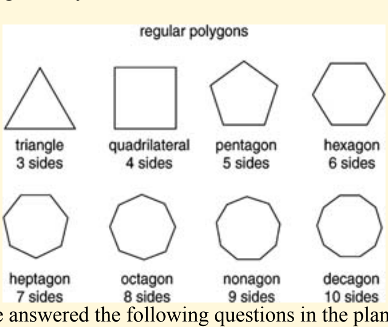
    Upshot: uniqueness is really helpful; **uniqueness is a big deal**.
- Last time we talked **symmetry**:

  Symmetry, as wide or as narrow as you devine its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection.

  Hermann Weyl (German Mathematician; 1885 - 1955)
  - You have been assigned parts of this worthy handout for homework (see the assignment page). It covers two kinds of symmetry that are very important: rotational and reflective.
    - Are humans symmetric? Humans are purported to have a "good side" and an "evil side"!
    - Symmetry appears everywhere in nature. We'll see three-dimensional versions of that today.

      But Weyl's classic *Symmetry* gives us some beautiful examples (e.g. pages 59, 75).
- Today we consider the Platonic solids possess a great deal of symmetry: they're super-symmetric solids.
  - We begin our discussion with the convex regular polygons:

    "A **regular polygon** is a polygon which is equiangular (all angles are equal in measure) and equilateral (all sides have the same length). Regular polygons may be convex or star."

  - Last time we answered the following questions in the plane of 2-Dimensions:
    - What is the most symmetric rectangle?
    - What are regular polygons?
    - How many regular polygons are there?
    - What kinds of symmetry do they possess?
  - Which regular polygons can be used to tile (tessellate) the plane?
  - What could you do with a heptagon?
  - As an example, consider regular convex hexagonal graph paper, which represents a **tiling of the plane** by a regular convex polygon. You see these kinds of tilings in bathrooms.

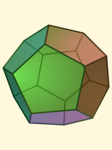
    Bees have the same idea:

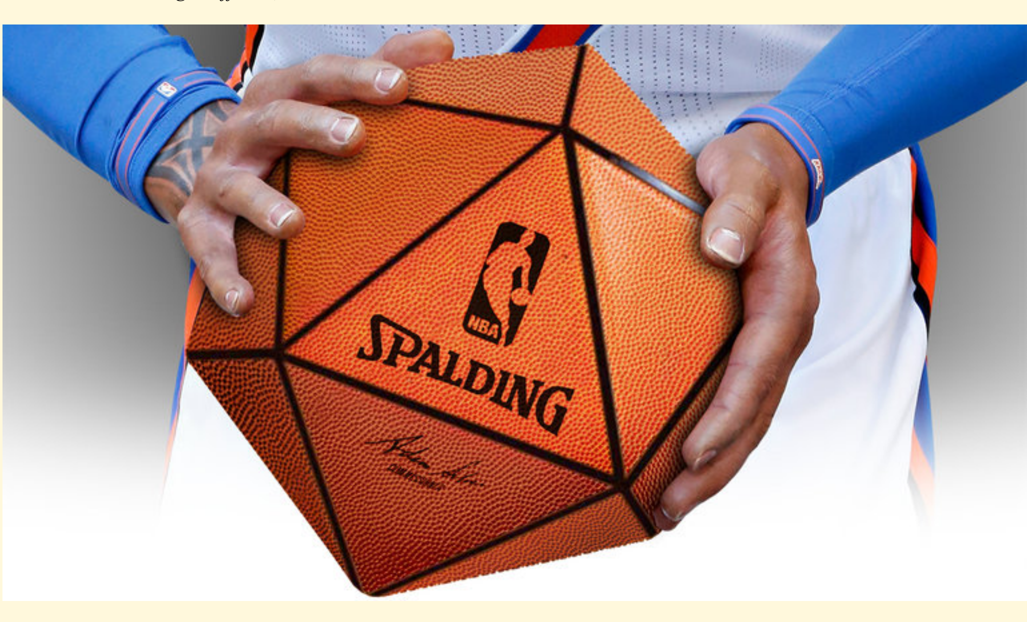
    
  - The honeycomb is actually solid -- three-dimensional. The tubes have hexagonal walls. But the comb is not Platonic.

    A cube is an example of a Platonic solid. It's the one we're most familiar with, so let's start with that.

    A Platonic solid is a solid for which
    - All faces are congruent (identical) convex regular polygons, and
    - Each face, each edge, and each vertex is exactly equivalent to every other face, edge, or vertex (respectively).

    The Five Convex Regular Polyhedra (Platonic solids) -- thanks Wikipedia!

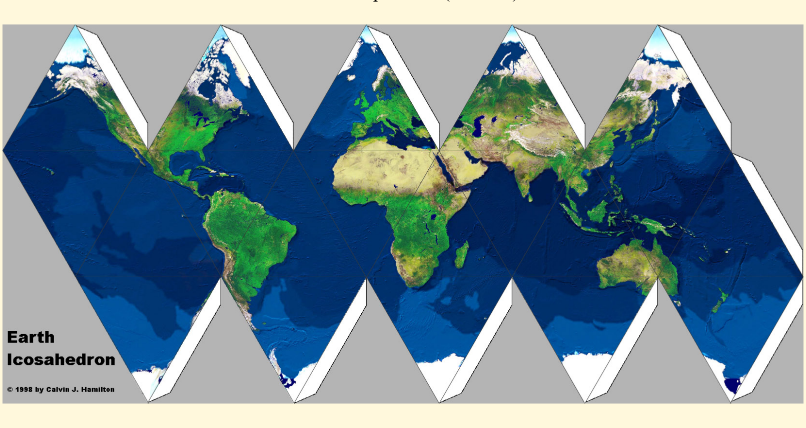
Tetrahedron	Hexahedron or Cube	Octahedron	Dodecahedron	Icosahedron
				
(Animation) fire	(Animation) earth	(Animation) air	(Animation) universe	(Animation) water
  - What do these have to do with Plato? According to Weyl, "Plato, in the dialogue *Timaeus,* associates the regular pyramid, octahedron, cube, icosahedron, with the four elements of fire, air, earth and water (in this order), while in the pentagondodecahedron he sees in some sense the image of the universe as a whole."
  - As a "homework" (for your own good!), I am asking you to cut and create Platonic solids out of paper, using this template. You may use these as a cheat sheet for the next exam. You must have put them together, however, and you must use only your own.
  - What's wrong with six sided polygons, and beyond? Consider the hexagonal graph paper, of the tiling of the plane: why can't it be folded up into a solid?
  - What can you do with these solids?
    - Play basketball? *Three Sides to This Story: Knicks Dust Off Phil Jackson's Triangle Offense*, from the New York Times:

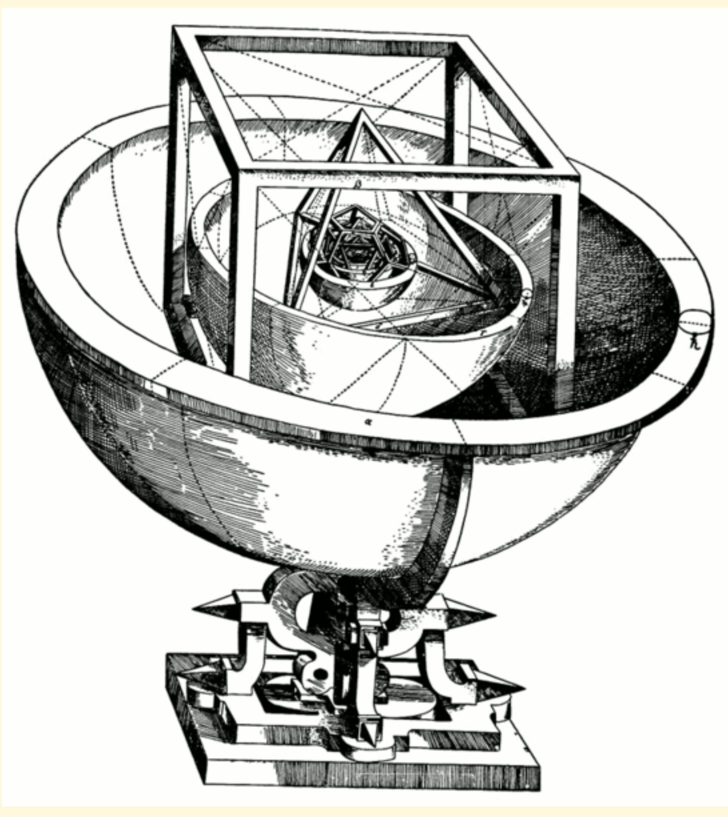
    - Map the Earth?

      The Platonic solids allow us to make "flat spheres" (sort of!):

    - Johannes Kepler thought that the orbits of the planets (of which five were known at the time) were somehow related to the various Platonic solids, as shown in

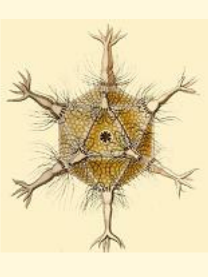
      

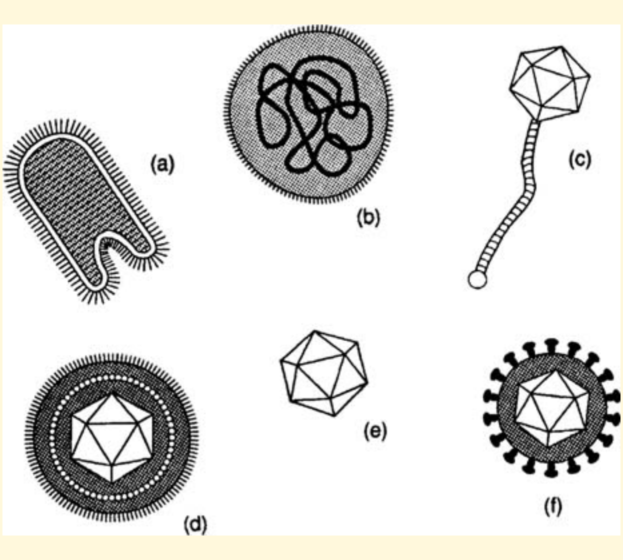
      "The six spheres each corresponded to one of the planets (Mercury, Venus, Earth, Mars, Jupiter, and Saturn). The solids were ordered with the innermost being the octahedron, followed by the icosahedron, dodecahedron, tetrahedron, and finally the cube. In this way the structure of the solar system and the distance relationships between the planets was dictated by the Platonic solids."

      And then Uranus was discovered, and there was not sixth Platonic solid. And so science evolves....
    - Something interesting to share: great images from Linus Pauling's and Roger Hayward's *The Architecture of Molecules* (a book created especially for young people, and based off of Pauling's article in the 1964 Proceedings of the National Academy of Sciences.

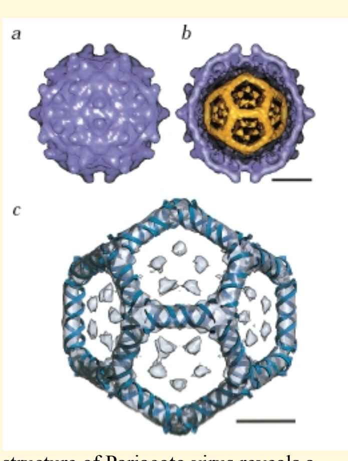
      These are some higher quality images than my scans:
      - Beautiful pictures!
      - More Beautiful pictures!
    - So what can you do with a dodecahedron? Look like a super-brainy, cool scientist!

      From the October 7th, 2011 New York Times
    - Icosahedron
      - amoeboid protozoa Circogonia icosahedra

        
      - Viruses:
        - The Cliff Notes Version...

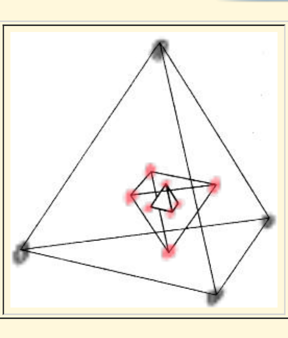
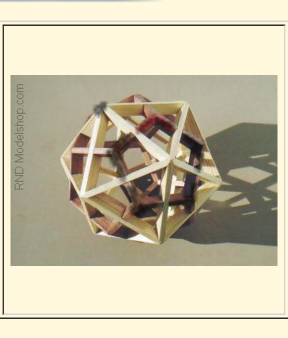
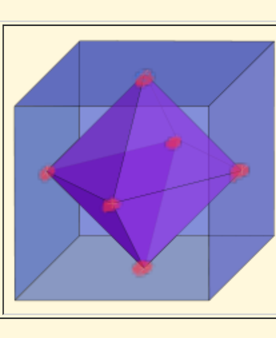
        "An array of viruses. (a) The helical virus of rabies. (b) The segmented helical virus of influenza. (c) A bacteriophage with an icosahedral head and helical tail. (d) An enveloped icosahedral herpes simplex virus. (e) The unenveloped polio virus. (f) The icosahedral human immunodeficiency virus with spikes on its envelope."
        - Bacteriophages and Virions
      - Dodecahedral
        - Viruses:

        "The structure of Pariacoto virus reveals a dodecahedral cage of duplex RNA"
    - More critters:
      - Canine Parvovirus
      - Enterobacteria Phage
  - Now let's use what we can deduce of Platonic solids to fill in the following table:

	# of Vertices	Edges	Faces	faces at each vertex	sides at each face
Tetrahedron	4	6	4	3	3
Cube	8	12	6	3	4
Octahedron	6	12	8	4	3
Dodecahedron	20	30	12	3	5
Icosahedron	12	30	20	5	3

    What conclusions can we draw from this data? Is there a pattern? (Of course there is!:) The pattern leads to the concept of "Duality":
    - Each Platonic solid has a "twin" -- called its dual -- which we can discover from the table.

Links:

- There are also "star Platonic solids", as we see in this lovely stellated dodecahedron.
- A lovely treatise on symmetry
- The "Spirograph" is a wonderful tool for making symmetric objects:
  - http://cgibin.erols.com/ziring/cgi-bin/spiro.pl/spiro.html
  - http://wordsmith.org/~anu/java/spirograph.html

Website maintained by Andy Long. Comments appreciated.
Updated on 03/08/2021 01:29:39