Practice for Exam 2, MAT115

The following problems appeared on my exams in the past, and are typical of those you might expect on your exam this week.

However, the problems are not **exhaustive** – these are not the **only** kinds of problems you might see. Examine your homework problems, do your readings, and expect some problems that may look like some you've never seen before (but which you could solve if you understand the concepts). Life doesn't just give you problems like those you've already encountered....

Problem numbering and points per problem reflect the exams from which I pulled these, and don't really mean anything here!

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A short 2/n unit fraction table from the Rhind Mathematical Papyrus:

2/3 = 1/2 + 1/6       2/5 = 1/3 + 1/15        2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18    2/11 = 1/6 + 1/66    2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30    2/17 = 1/12 + 1/51 + 1/68    2/19 = 1/12 + 1/76 + 1/114
2/21= 1/14 + 1/42    2/23 = 1/12 + 1/276    2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54    2/29 = 1/24 + 1/58 + 1/174 + 1/232      2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66    2/35 = 1/30 + 1/42    2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78    2/41 = 1/24 + 1/246 + 1/328    2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90    2/47 = 1/30 + 1/141 + 1/470    2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102    2/53 = 1/30 + 1/318 + 1/795    2/55 = 1/30 + 1/330
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Problem 2: (10 pts) Symmetry:

a. I've provided space below the figure for your answers: please put them there.

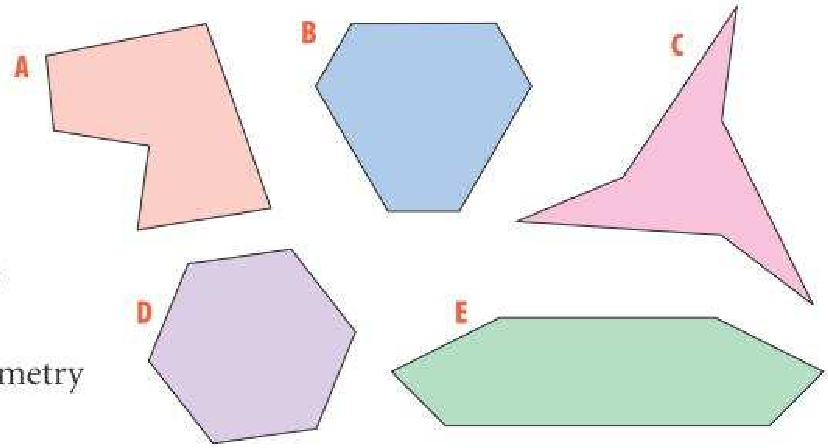
All these shapes are hexagons.
Which hexagon has

(a) only one line of symmetry

(b) rotation symmetry but no reflection symmetry

(c) rotation symmetry of order 3 and 3 lines of symmetry

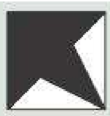
(d) no reflection or rotation symmetry



- (a)
- (b)
- (c)
- (d)

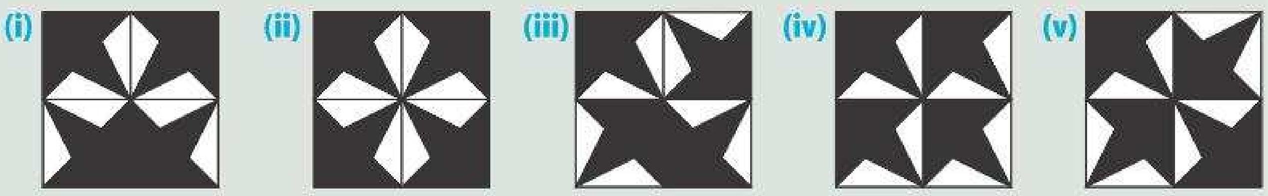
b. For the following problem, write your answer directly under each pattern in part (a); for (b), just fill in the proper pattern. If you mess up, draw another elsewhere on the paper.

Judith has lots of tiles, all like this one.

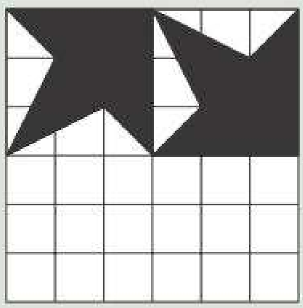


(a) Judith makes these patterns.

For each pattern, write down the number of lines of symmetry it has.
If the pattern does not have reflection symmetry, write 0.



(b) Copy and complete this tiling pattern so that it has rotation symmetry of order 4.



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Problem 3: (10 pts) Short answer: Readings

a. In your readings you discovered the importance of Arabic mathematicians. Demonstrate some familiarity with their contributions to mathematics. You might say something about Omar Khayyam, for example. What did he do that was related to our course?

b. To what does "spin in the spring, flip in the fall." refer?

c. To what does "The Enemy of my Enemy is my Friend" refer?

d. There was one "non-unit" fraction that the Egyptians permitted – what was it?

Problem 4: (10 pts) The golden ratio ϕ was discovered in class in two different ways: from ratios of Fibonacci numbers, and by chopping up a beautiful rectangle and using the quadratic formula.

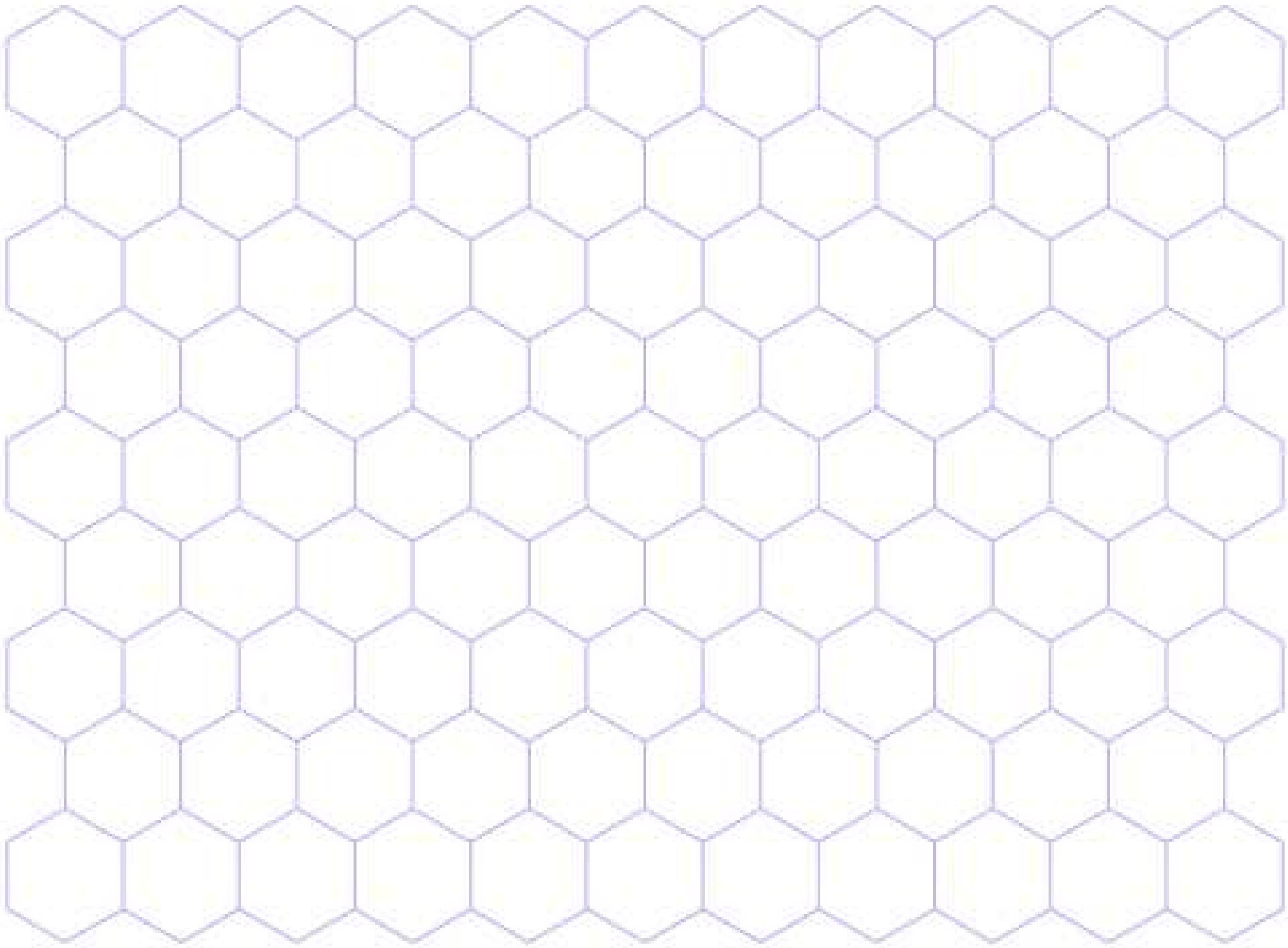
a. (2 pts) What is the true value of ϕ? What is its best four digit approximate value?

b. (4 pts) What was the Greek's definition of a golden rectangle, and how does it relate to ϕ? **Note:** you do not need to **derive** ϕ: just explain how the Greek's went about defining it (perhaps with a diagram).

In the grid provided, make the largest Fibonacci spiral you can. At right, describe the connection between this spiral process and the golden ratio and rectangle.

At each stage, compute the ratio of the largest to the smallest side. What special value are these ratios approaching as the rectangles increase in size?

Problem 5: (10 pts) Pascal's triangle (4 pts) Use this hexagonal grid to create Pascal's triangle, starting down from a "1" in the top row, center: As part of your construction, illustrate how the triangle contains



each of the following in a systematic way:

a. The Powers of 2

b. The Triangular numbers

c. The Fibonacci numbers

Problem 6: (10 pts) Demonstrate Egyptian Multiplication by multiplying

a. 39*63

b. 81*115

Problem 7: (10 pts) Demonstrate Egyptian division (give your answer as Egyptians would) for the following. You may use either of our two methods (the unit fraction table – there's a table at the end of the test – or the doubling/halving table).

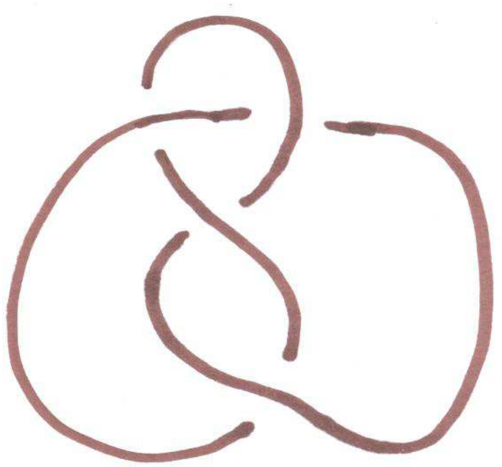
a. Compute $\frac{21}{32}$.

b. Divide 9 loaves among 11 people.

Problem 3: Here's an unusual floor plan for a house with very curvy walls (gaps are "doors"):

Is it possible to pass through each door once without passing through any door twice?

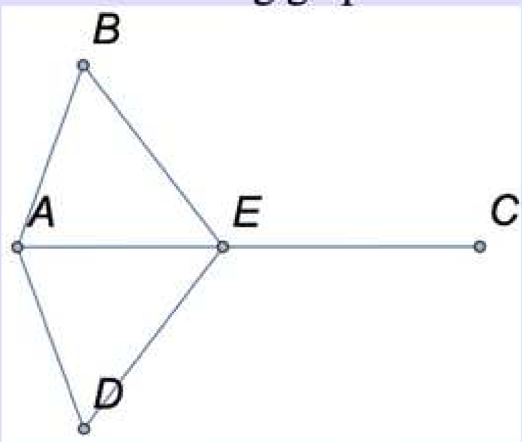
What about the floor plan for the figure-eight knot (same question as above):



Problem 8: Draw all simple graphs with four vertices. Indicate which graphs are duals of each other.

Problem 9: Carefully draw either one of the two simplest **non-planar** graphs. What does it mean for a graph to be non-planar?

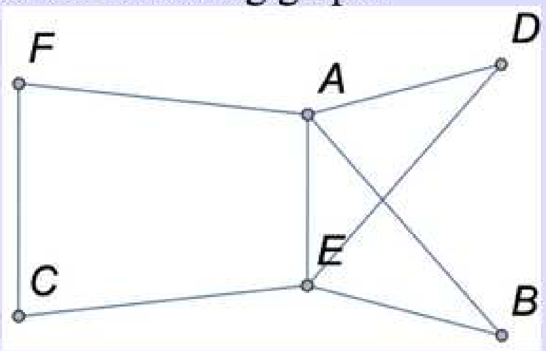
Consider the following graph.



1. What is the degree of each vertex?
   a. $deg(A) =$
   b. $deg(B) =$
   c. $deg(C) =$
   d. $deg(D) =$
   e. $deg(E) =$
2. If it has an Euler path starting at E, give its path as a list of the vertices. If it does
   not exist."

   Euler path:

Consider the following graph.



1. What is the degree of each vertex?
   a. $deg(A) =$
   b. $deg(B) =$
   c. $deg(C) =$
   d. $deg(D) =$
   e. $deg(E) =$
   f. $deg(F) =$
2. If it has an Euler circuit starting and ending at A, give its path as a list of the vertice enter DNE for "does not exist."

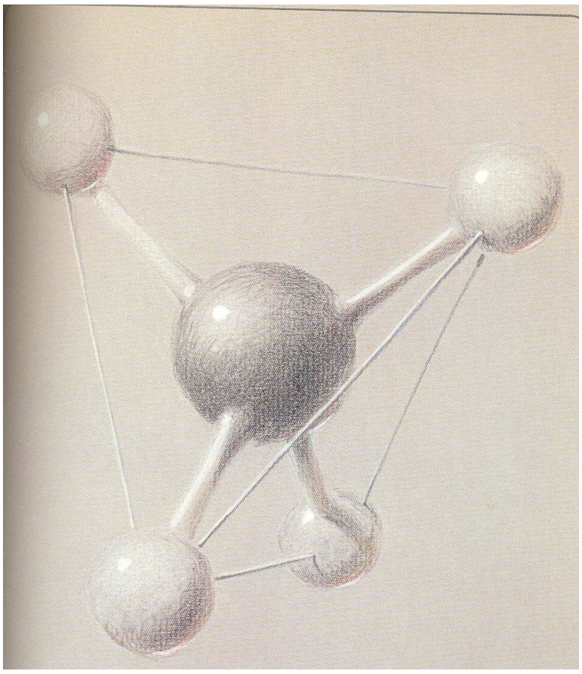
   Euler circuit:

Problem 10: What does it mean if I say that the graph of Facebook relations between five particular people is a complete simple graph?

Problem 14: Use Egyptian Multiplication to multiply 33*59.

Problem 15: Use Egyptian division to divide 7 loaves among 9 people, writing the answer as the ancient Egyptians did. You may use either of our two methods (the unit table – there's a table at the end of the test – or the doubling/halving table).

Problem 18: At left is methane, a potent greenhouse gas. What's Platonic about it?



At right of the methane molecule, draw a hexahedron and its dual.

Problem 20: Platonic solids. Complete the following table for the five Platonic solids:

Solid Name	# of vertices	# of edges	# of faces	# edges per face	# edges per vertex
T					
C					
O					
I					
D					

Use this table to describe which solids are duals with which other solids, and why.

Problem 5: (10 pts) Short answer:

a. Why do we suppose that the ancient Egyptians only liked unit fractions? What was the prevailing theory discussed in class?

b. What is the Rosetta stone, and why was it so important?

c. If there is an Euler path in a connected graph, how many vertices may have odd degrees? (Give a complete answer: it's an either/or, not just a single number.)

Problem 7: (20 pts) Egyptian Math

a. Use Egyptian Multiplication to multiply 85*143.

b. Divide 13 loaves among 15 people, writing the answer as the ancient Egyptians did. You may use either of our two methods (the unit table, or the doubling/halving table), although I suggest the doubling/halving table.

Problem 3 (8 points): Describe at least two specific examples of where we find the following in nature:

a. Platonic solids

b. Fibonacci numbers

Problem 4 (8 points):

a. (5 pts) What is the definition of a Platonic solid? What must be true of the faces, edges, and vertices?

b. (5 pts) Name all the Platonic solids.

c. (3 pts) Which are duals of which, and why?

d. (3 pts) Write down the first 15 Fibonacci numbers in order.

Problem 8 (10 points):

a. How did the Greeks define a golden rectangle?

b. Given the following rectangles, which one is closest to golden? Give evidence....

  i. A 3x5 card
  ii. A sheet of $8\frac{1}{2}$ by 11 paper
  iii. A flag of dimensions 4 feet by 6 feet
  iv. A television screen of size 30x48 inches?

Problem 6: (20 pts) Egyptian Math

a. Use Egyptian Multiplication to multiply 45*122.

b. Divide 4 loaves among 15 people, writing the answer so as to make the ancient Egyptians happy (using only unit fractions). You may use either of our two methods (the unit table, or the doubling/halving table).