

$$g(x) = \ln(x) = \ln(9 + (x-9))$$

related to integration + geometric

series

$$g'(x) = (\ln(x))' = \frac{1}{x} = \frac{1}{9 + x - 9}$$

$$= \frac{1}{9} \left(\frac{1}{1 + \frac{x-9}{9}} \right)$$

$$= \frac{1}{9} \left(\frac{1}{1 - \underbrace{\left(-\frac{(x-9)}{9} \right)}_r} \right) = \frac{1}{9} \cdot \frac{1}{1-r}$$

$$= \frac{1}{9} \sum_{n=0}^{\infty} r^n \quad \checkmark \quad \text{where } r = \frac{-(x-9)}{9}$$

$$g'(x) = \frac{1}{9} \sum_{n=0}^{\infty} \left(-\frac{(x-9)}{9} \right)^n = \frac{1}{9} \sum_{n=0}^{\infty} \frac{(-1)^n (x-9)^n}{9^n}$$

~~series~~

$$g(x) = \int g'(x) dx = \frac{1}{9} \sum_{n=0}^{\infty} \frac{(-1)^n}{9^n} \int (x-9)^n dx$$

$$= \frac{1}{9} \sum_{n=0}^{\infty} \frac{(-1)^n}{9^n} \frac{(x-9)^{n+1}}{n+1} + C$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{9^{n+1}} \frac{(x-9)^{n+1}}{n+1} + \ln 9$$