Review for “New Stuff” final

Exam rules

- You can have one page of notes (front and back) and you are allowed to have a calculator.
- You have the full two-hour period to work on it. The exam is on Canvas.
- You must show full work on all integrals.

Format

- Questions will be similar to daily homework questions and weekly assignment questions.

Topics

- Section 10.1: Parametric equations
  - Be familiar with techniques for changing parametric equations $x = x(t)$, $y = (t)$ into a single Cartesian equation.
  - Find parametric equations for circles.
- Section 10.2: Calculus with parametric equations
  - Slopes and tangent lines.
  - Horizontal and vertical tangents.
  - Length of curves parametrically defined.
- Section 10.3: Polar coordinates
  - Convert between Cartesian coordinates and polar coordinates.
  - Knowing the simpler equations in polar coordinates.
  - Polar curves $r = f(\theta)$.
  - Writing polar curves as parametric equations to find slopes and lengths.
  - Finding periods of polar curves.
- Section 10.4: Areas of polar curves
  - Differences of areas to find area between curves.
  - Finding intersections of curves.
- Section 12.1: 3D coordinates

    - Distance formula.
    - Equations for simple objects (spheres, planes parallel to coordinates axes, etc.).
- Section 12.2: Vectors
    - Significance of vector components.
    - Magnitude.
    - Direction of 2D vectors as an angle with the positive *x*-axis.
    - Scalar multiplication and its geometric significance.
    - Vector addition and its geometric significance.
    - Unit vectors i, j, k. Orthogonal coordinate systems. Frenet Frame.
- Section 12.3: Dot product
    - Find angle between two vectors.
    - Test if two vectors are perpendicular to each other.
    - Dot products relationship to magnitude.
    - Vector projections.
- Section 12.4: Cross product
    - Only defined for two space vectors.
    - $\vec{u} \times \vec{v}$ is perpendicular to both $\vec{u}$ and $\vec{v}$.
    - $\vec{u} \times \vec{v}$ is a good test of when vectors $\vec{u}$ and $\vec{v}$ are parallel (the cross product is the 0 vector).
    - $|\vec{u} \times \vec{v}|$ is the area of the parallelogram formed from $\vec{u}$ and $\vec{v}$.
    - $|\vec{u} \times \vec{v}|$ is twice the area of the triangle formed from $\vec{u}$ and $\vec{v}$.
    - Derivation of Kepler's First Law.

Studying

- Try problems you haven't worked before from the exercises in your reference textbook.
- Look back through the weekly homework assignments.
- Rework problems in IMath.

Sample questions

1. Consider the parametric equations $x = 4\sin(\pi t)$, $y = 4\cos(\pi t) + 1$.

1.1. Find an equation for its tangent line at $t = 1/4$.

1.2. Find all points on the curve that have horizontal tangents.

1.3. Eliminate the parameter to find a Cartesian equation of the curve.

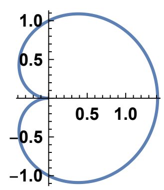
2. Polar coordinates for a point are $(r,\ \theta) = (4,\ \pi/3)$.

2.1. Plot the point in the plane.

2.2. Find two other different polar coordinates for this same point, one with $r < 0$ and one with $r > 0$ but a different value for θ.

2.3. Give Cartesian coordinates (x, y) for this point.

3. What is the area enclosed by the polar curve $r = \sqrt{\cos(\theta) + 1}$, $0 \le \theta \le 2\pi$, shown below.



4. Find the slope of the tangent line to the polar curve $r = \theta$ when $\theta = \pi/4$.

5. Find a Cartesian equation for the curve given by $r = 2\cos(\theta)$.

6. Find an equation for the sphere with center $(2, -3, 6)$ that just touches the x-y plane.

7. Find a unit vector that points in the same direction as the vector that points from point $(1, 0, -2)$ to point $(3, 1, 1)$.

8. Find the values of x such that the angle between the vectors $\langle 2, 1, -1\rangle$ and $\langle 1, x, 0\rangle$ is 45°.

9. Consider the two vectors $\vec{u} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{v} = -2\,\vec{i} + 3\,\vec{k}$.

9.1. What is the area of the parallelogram whose sides are formed from $\vec{u}$ and $\vec{v}$?

9.2. Find two different unit vectors that are orthogonal to $\vec{u}$ and $\vec{v}$.

10. Find an equation for the plane that is parallel to $3x + 2y + z = 20$ and that passes through the point $(1, 0, -1)$.