Section Summary 11.6: Absolute versus conditional convergence, root and ratio tests

1 Definitions

Absolutely convergent: A series is called absolutely convergent if the series of absolute values $\sum |a_n|$ converges.

Conditionally convergent: A series is called conditionally convergent if the series is **convergent but not absolutely convergent**.

2 Theorems

If a series $\sum a_n$ is absolutely convergent, then it is convergent.

The ratio test:

a. If
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1,$$
then the series $\sum a_n$ is absolutely convergent.

b. If
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1,$$
(or $L = \infty$) then the series $\sum a_n$ is divergent.

The root test:

a. If
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1,$$
then the series $\sum a_n$ is absolutely convergent.

b. If
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1,$$
(or $L = \infty$) then the series $\sum a_n$ is divergent.

3 Properties, Hints, etc.

One of the really freaky things about conditionally convergent series is that re-arrangement of terms results in any sum one desires. (This is not the case with absolutely convergent series).

4 Summary

The difference in the ratio and root tests in this section is that they are "self-referential": we compare terms of a_n with "themselves" (the root test) or with other another term in the sequence, a_{n+1}.