Lab 11: Series Review

Geometric Series:

$$S = \sum_{k=0}^{\infty} a(r)^k \quad \text{Converges if } |r| < 1$$

$$P = 1 + r + r^2 + \dots + r^N$$

$$rP = r + r^2 + \dots + r^N + r^{N+1}$$

$$rP - P = r^{N+1} - 1$$

$$(r-1)P = r^{N+1} - 1 \qquad \therefore P = \frac{r^{N+1} - 1}{r - 1}$$

If $|r| < 1$ $\quad \lim_{N \to \infty} P = \frac{-1}{r-1} = \frac{1}{1-r}$

$$S = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$$

$$|S - S_N| = \sum_{k=N+1}^{\infty} ar^k$$

$$= ar^{N+1} + ar^{N+2} + \dots$$

$$= ar^{N+1}\left[1 + r + \dots\right] = \frac{ar^{N+1}}{1-r}$$

p-series: $\sum_{k=1}^{\infty} \frac{1}{k^p}$

Converges if $p > 1$

Harmonic Series: $\sum_{k=1}^{\infty} \frac{1}{k}$

diverges, ever so slowly.

Alternating Series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$$

$$s = \sum_{k=1}^{\infty} (-1)^{k+1} b_k$$

Converges if $b_k > 0$, are decreasing, & $\lim_{k \to \infty} b_k = 0$.

$$|s - s_n| = \left| \sum_{k=n+1}^{\infty} (-1)^{k+1} b_k \right| \le b_{n+1}$$