Lab 14

MAT 229, Spring 2021

1. Identifying curves. For each of the next three sets of parametric equations,

- Eliminate the parameter to get a Cartesian equation.
- Graph and identify the curve. You may graph parametrically, or using the Cartesian equation.

1.1. $x = \cos(t)$

$y = \sec(t)$

1.2. $x = 2\, e^{t} + 3$

$y = -e^{2\, t}$

1.3. $x = 10 \sin(3\, t) + 100$

$y = 10 \cos(3\, t) + 200$

2. The Bowditch curve is given parametrically by $x = \sin(t/2)$, $y = \sin(t)$.

2.1. It is a closed curve meaning it begins to repeat itself as t increases. Starting with $t = 0$, find a positive value of t where it begins to repeat. Then have Mathematica plot the curve.

2.2. From your plot you should see that there are four places where the curve has horizontal tangents, $\frac{dy}{dx} = 0$. Find the parameter values as well as the coordinates for these four points.

2.3. From your plot you should see that there are two places where the curve has vertical tangents, $\frac{dy}{dx}$ undefined. Find the parameter values as well as the coordinates for these two points.

3. The Cycloid curve is given parametrically by $x=2\pi t-\sin(2\pi t), y=1-\cos(2\pi t)$. It can be thought of as the movement of a point on the outside of a bike tire, of radius 1. Every now and then the point on the tire meets the road....

3.1. What is the period of this curve? Graph this curve over two periods.

3.2. Plot the parametric curve of the derivatives: x'(t),y'(t). What is its period?

3.3. Can you relate the derivative curve to the motion of the bicycle tire? Check out this manipulate command, and explain the relationship between the two curves.

In[∘]:=
```
x[t_] := 2 Pi t - Sin[2 Pi t]
y[t_] := 1 - Cos[2 Pi t]
Manipulate[
 Show[
  ParametricPlot[{x[t], y[t]}, {t, 0, 2}, PlotStyle → {Red, Thick}],
  ParametricPlot[{x'[t], y'[t]}, {t, 0, 1}, PlotStyle → {Black, Thick}],
  ListPlot[{{x[t], y[t]}}, PlotMarkers → {
```

```
  }],
  ListPlot[{{x'[t], y'[t]}}, PlotMarkers → {
```

```
  }],
  PlotRange -> All
 ], {t, 0, 2}]
```