Lab 15

MAT 229, Spring 2021

Polar curves

Given an equation on r and θ, all the points whose polar coordinates satisfy the equation form a curve, a polar curve. Graphing calculators and software can plot curves given by $r = f(\theta)$.

To plot the polar curve $r = f(\theta)$, $a \le \theta \le b$, in *Mathematica*, use the command **`PolarPlot`**.

Example

To plot $r = 1 - 2\cos(\theta)$ for $0 \le \theta \le 2\pi$, enter

```
PolarPlot[1 - 2 Cos[t], {t, 0, 2 Pi}]
```

Questions

1. The polar curve $r = 1 + 2\cos(\theta)$.

1.1. This curve is periodic. What is its period? (Give your answer as an interval, like *[0,p)* where p is the period.)

1.2. Plot the curve on this interval.

1.3. For which values of θ is $r < 0$?

1.4. Extremes of r:

1.4.1. What is the largest possible value of r?

1.4.2. What is its minimum value?

1.4.3. What are the corresponding θ-values and the points in Cartesian coordinates?

1.5. What are the values of θ at the point of self-intersection for this curve?

2. The polar curve $r = 1 - 2\cos(\theta)$ is closely related to $r = 1 + 2\cos(\theta)$. Answer the same questions for $r = 1 - 2\cos(\theta)$ that you answered for $r = 1 + 2\cos(\theta)$.

2.1. This curve is periodic. What is its period? (Give your answer as an interval, like *[0,p)* where p is the period.)

2.2. Plot the curve on this interval.

2.3. For which values of θ is $r < 0$?

2.4. Extremes of r:

2.4.1. What is the largest possible value of r?

2.4.2. What is its minimum value?

2.4.3. What are the corresponding θ-values and the points in Cartesian coordinates?

2.5. What are the values of θ at the point of self-intersection for this curve?

2.6. How are the two curves related?

3. Consider now the polar curve $r = 1 + 2\sin(\theta)$.

3.1. How is this curve $r = 1 + 2\sin(\theta)$ related to $r = 1 + 2\cos(\theta)$?

3.2. What are their points of intersections?

3.3. Compute the length of this curve over one period (write the proper integral, which you may compute numerically).

3.4. Compute the area of the inside loop (write the proper integral, which you may compute numerically).

4. The polar curve $r = \sin(n\,\theta)$ where n is a whole number is called a rose.

4.1. For each of $n = 1, 2, 3, 4$ determine the range of values of θ needed to draw the curve until it starts repeating itself. Make a conjecture about the range of values of θ needed for general n; make sure to test your conjecture.

4.2. Plot this curve for several different values of n and make (and test) a conjecture about the number of petals for the rose $r = \sin(n\,\theta)$.

4.3. For any given n, what range of values of θ are needed to draw one petal of the rose?