Lab 16: Vectors and Vector Products

MAT 229, Spring 2021

Vector review

A vector is an object that has magnitude and direction.

Vectors in the plane

- Can be represented in component form (Cartesian coordinates) as $\vec{v} = \langle a, b \rangle$.
- Can be represented in magnitude, angle form (polar coordinates) as $\left| \vec{v} \right| = \sqrt{a^2 + b^2}$, and $\tan(\theta) = \frac{b}{a}$ where θ is the angle between the positive x-axis and the vector.

Vectors in space

- Can be represented in component form (Cartesian coordinates) as $\vec{v} = \langle a, b, c \rangle$.

Dot product review

There are two ways to view the dot product of two vectors:

- If $\vec{u} = \langle a, b, c \rangle$ and $\vec{v} = \langle \alpha, \beta, \gamma \rangle$, then $\vec{u} \cdot \vec{v} = a \times \alpha + b \times \beta + c \times \gamma$.
- If θ is the angle between $\vec{u}$ and $\vec{v}$, then $\vec{u} \cdot \vec{v} = \left| \vec{u} \right| \left| \vec{v} \right| \cos(\theta)$.

Unit vector review

Definition

A *unit vector* is a vector with magnitude 1. Its magnitude is fixed, but its direction can be any direction.

Problems to submit

1. Let $\vec{u} = \langle 1, 1 \rangle$ and $\vec{v} = \langle -2, 1 \rangle$.

1.1. Find the magnitudes of both.

1.2. Find the angle each makes with the positive x-axis.

1.3. Draw vectors $\vec{u}$, $\vec{v}$, $\vec{u} + \vec{v}$, and $\vec{u} - \vec{v}$ in the x-y plane.

2. Let $\vec{v} = \langle 1, 1, 2 \rangle$,

2.1. Find a unit vector that points in the same direction as $\vec{v}$ by multiplying $\vec{v}$ by an appropriate scalar.

2.2. Find a unit vector that points in the opposite direction as $\vec{v}$.

2.3. Find a vector that has length 4 and points in the same direction as $\vec{v}$.

3. A treasure hunt has the following instructions:

- At the starting point head 20 yards 30° north of east.
- At the new point head 30 yards due north.
- At this location head 40 yards northwest to the point where the treasure is located.

3.1. Write each instruction as a vector in component form.

3.2. Draw a graph of these vectors so that for each vector after the first is drawn with its initial point at the terminal point of the previous vector.

3.3. Find a vector whose initial point is at the starting point of the treasure hunt and whose terminal point is the treasure's destination.

4. A woman walks due west on the deck of a ship at 3 mph. The ship is moving north at a speed of 22 mph. Find the speed and direction of the woman relative to the surface of the water.

5. Two nonzero vectors are perpendicular if and only if their dot product is zero.

5.1. Use this to find a unit vector $\langle x, y \rangle$ that is perpendicular to vector $\langle 4, 3 \rangle$ by writing two equations that x and y must satisfy. Don't forget being a unit vector puts an equation on x and y. There are two such vectors, can you find both?

5.2. Use this same idea to find a unit vector $\langle x, y, z \rangle$ that is perpendicular to both vector $\langle 2, 0, 1 \rangle$ and vector $\langle 0, -2, 1 \rangle$. There are two such vectors, can you find both?

6. Tangent lines have directions, so vectors can be parallel or perpendicular to them.

6.1. What angle does the tangent line to $y = \sin(x)$ at $x = \pi$ make with the x-axis? Find the two unit vectors parallel to this tangent line.

6.2. Find the unit vectors that are perpendicular to the tangent line to $y = \sin(x)$ at $x = \pi/4$.