Lab 2: Student Assignment

Week 2, January 18-24

MAT 229, Spring 2021

Special Constants

Standard notation	Mathematica notation
$\pi \approx 3.14159$	**Pi**
$e \approx 2.71828$	**E**

Commands

Functionality	Mathematica notation
plot the graph of a function	**Plot[...]**
square root of something, $\sqrt{\ldots}$	**Sqrt[...]**
absolute value of something, $\lvert \ldots \rvert$	**Abs[...]**
sine of something (radian mode), sin(…)	**Sin[...]**
cosine of something (radian mode), cos(…)	**Cos[...]**
tangent of something (radian mode), tan(…)	**Tan[...]**
natural logarithm of something, ln(…)	**Log[...]**

Exercises to submit

Exercise 1

The linear approximation for a function $f(x)$ at $x = a$ is the $m\,x + b$ that comes from the tangent line $y = m\,x + b$ to $f(x)$ at $x = a$. It provides a simple approximation to $f(x)$ for values of x near a.

$$f(x) \approx m\,x + b$$

Let $f(x) = 2^x - 2\,x + 3$.

- Define this function in Mathematica.

```
f[x_] := 2^x - 2 x + 3
```

Now let’s use this function to create the tangent line at a given value x=x0. (Notice that the semi-colon suppresses reporting of the value of x0):

In[103]:=
```
x0 = 0;
f'[x0]
f[x0]
f'[x0] (x - x0) + f[x0]
```

Out[104]= -2 + Log[2]

Out[105]= 4

Out[106]= 4 + x (-2 + Log[2])

- Determine the linear approximation for $f(x)$ at $x = 0$.

  $m\,x + b =$ 4+x (-2+Log[2])

- What is the absolute value (decimal) of the difference between $f(1)$ and your linear approximation at $x = 1$?

Let' s use our powers as mathematicians to name something :

In[109]:=
```
tangent[x_] := 4 + x (-2 + Log[2])
Abs[f[1] - tangent[1]]
N[%]
```

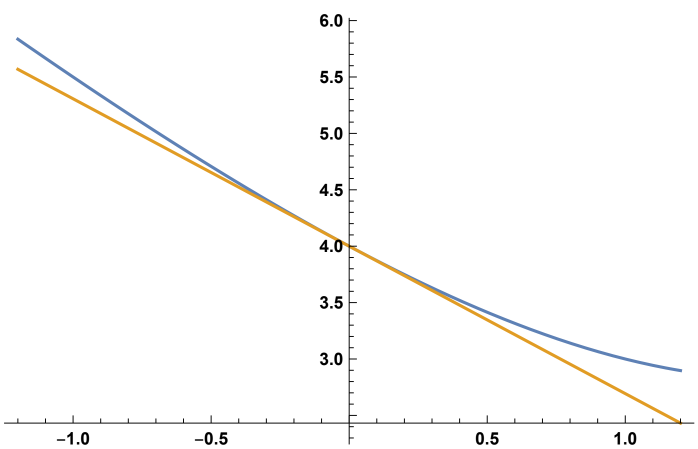
Out[110]= 1 - Log[2]

Out[111]= 0.306853

and let’s check that things make sense:

In[114]:=
```
Plot[{f[x], tangent[x]}, {x, -1.2, 1.2}]
```

Out[114]=



- What is the absolute value (decimal) of the difference between $f(-1)$ and your linear approximation at $x = -1$?

In[112]:=
```
Abs[f[-1] - tangent[-1]]
N[%]
```

Out[112]= $-\frac{1}{2} + \text{Log}[2]$

Out[113]= 0.193147

Exercise 2

Let $f(x) = \frac{e^{-x^2-x-1}}{x^2+1}$.

- Define this function in Mathematica.

Note that for this one I defined the function as “f2” -- if you want to avoid collisions, you can give each function in a notebook a unique name. Remember your power as a mathematician -- to name things!

```
f2[x_] := E^(-x^2 - x - 1) / (x^2 + 1)
```

- Use your function to determine the y-intercept of $y = f(x)$. Get a decimal value.

```
N[f2[0]]
```

- Get all real-valued critical numbers for $f(x)$ as decimal values.

In[117]:=
```
NSolve[f2'[x] == 0, x, Reals]
```

Out[117]= {{x → -0.258056}}

- Evaluate $f(x)$ at your critical numbers to determine the y-values of the critical points.

In[118]:=
```
f2[-0.25805587247847267`]
```

Out[118]= 0.417694

- Evaluate $f''(x)$ at your critical numbers to determine if each is a local maximum or a local minimum.

In[119]:=
```
f2''[-0.25805587247847267`]
```

Out[119]= -1.52082

Local minimum points: none

Local maximum points: (-0.25805587247847267, -1.52082)

- Get all real-valued second-order critical numbers for $f(x)$ as decimal values, i.e. solve $f''(x) = 0$. These are points about which concavity can change, the inflection points.

In[120]:=
```
NSolve[f2''[x] == 0, x, Reals]
```

Out[120]= {{x → -0.775715}, {x → 0.236704}}

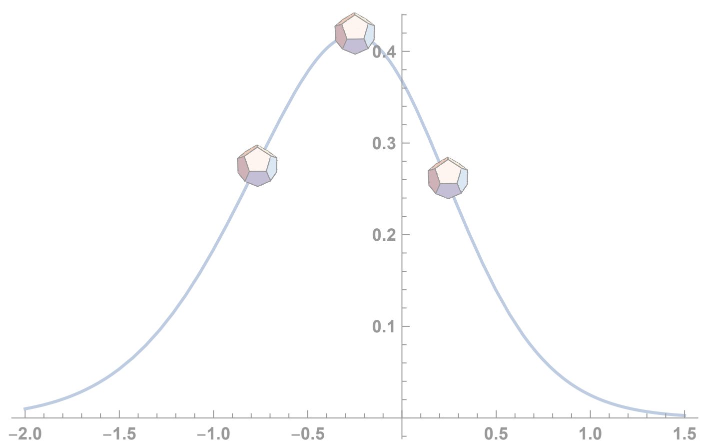
Inflection points:

- Make a plot of the graph of $f(x)$ to verify your calculations for the y-intercept, the local max/min, and the inflection points.

I think that I’ll have a little fun here:

```
Show[
 Plot[f2[x], {x, -2, 1.5}],
 ListPlot[
  {
   {-0.775714860650307`, f2[-0.775714860650307`]},
   {0.23670380674925937`, f2[0.23670380674925937`]},
   {-0.25805587247847267`, f2[-0.25805587247847267`]}
  }, PlotMarkers → { }]
]
```

Out[125]=



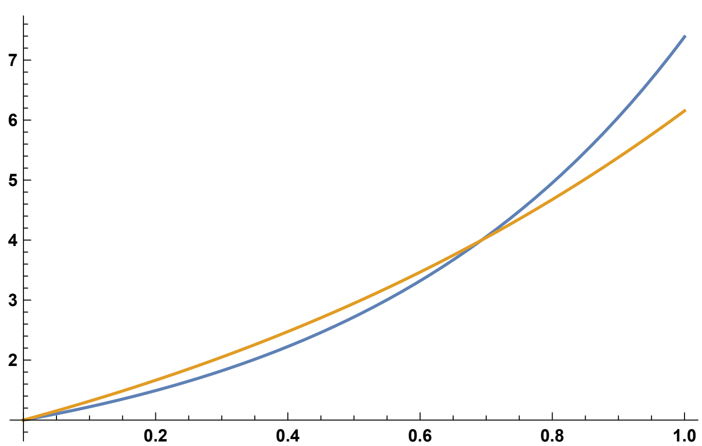
Exercise 3

Let $g(x) = e^{2x}$ and $h(x) = 3\,e^{x} - 2$.

- Plot the graphs of $g(x)$ and of $h(x)$ together on the same axes for various ranges of x-values until you have one that clearly shows the region bounded by these two graphs.

In[132]:=
```
g[x_] := E^(2 x)
h[x_] := 3 E^x - 2
Plot[{g[x], h[x]}, {x, 0, 1}]
```

Out[134]=



- Get decimal numbers for the x-values of the intersection points.

  Intersection points: $x \approx 0, 0.693147$

In[136]:=
```
NSolve[g[x] == h[x], x, Reals]
```
Out[136]=
```
{{x → 0.}, {x → 0.693147}}
```

- Find the area of the region bounded by $y = g(x)$ and $y = h(x)$.

  Area: 0.113706

We can see that h is on top, because ultimately g has to grow faster!

In[137]:=
```
NIntegrate[h[x] - g[x], {x, 0, 0.6931471805599453`}]
```
Out[137]=
```
0.113706
```

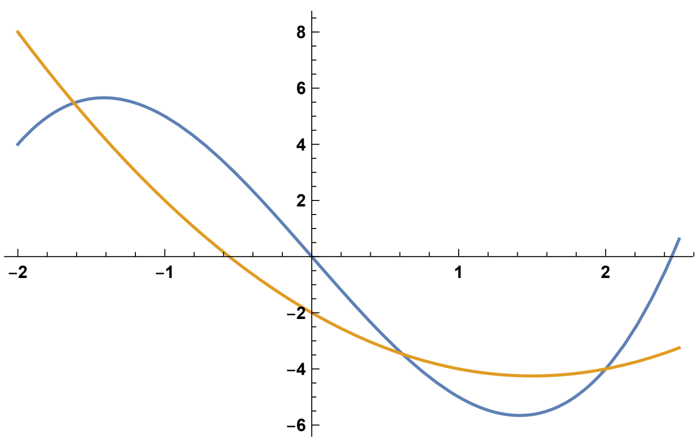
Exercise 4

Let $g(x) = x^3 - 6x$ and $h(x) = x^2 - 3x - 2$.

In[154]:=
```
g[x_] := x^3 - 6 x
h[x_] := x^2 - 3 x - 2
```

- Plot the graphs of $g(x)$ and of $h(x)$ together on the same axes for various ranges of x-values until you have one that clearly shows the regions bounded by these two graphs.

In[157]:=
```
Plot[{g[x], h[x]}, {x, -2, 2.5}]
```
Out[157]=



- Get exact numbers for the x-values of the intersection points.

  Intersection points: $x = 2, \frac{1}{2}\left(-1-\sqrt{5}\right), \frac{1}{2}\left(-1+\sqrt{5}\right)$

In[159]:=
```
Solve[g[x] == h[x], x]
```
Out[159]= $\left\{\{x \to 2\}, \left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

- Find the combined area of the regions bounded by $y = g(x)$ and $y = h(x)$.

  Area: 5.94605

In[161]:=
```
Integrate[Abs[g[x] - h[x]], {x, 1/2 (-1 - √5), 2}]
N[%]
```

Out[161]= $\frac{25}{24}\left(-1+3\sqrt{5}\right)$

Out[162]= 5.94605

Exercise 5

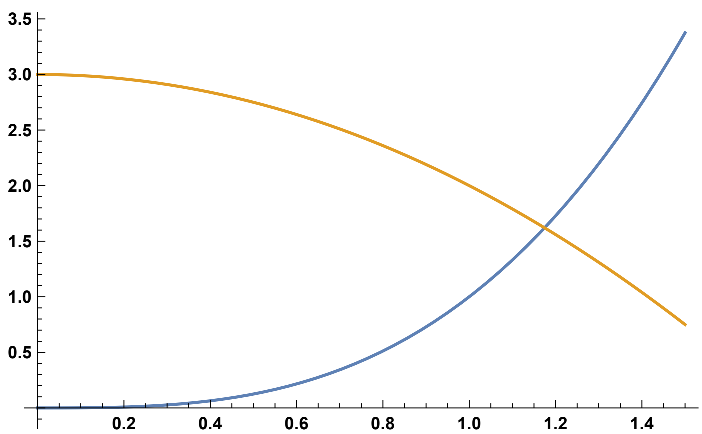
Let $p(x) = x^3$ and $q(x) = -x^2 + 3$.

In[163]:=
```
p[x_] := x^3
q[x_] := -x^2 + 3
```

- Plot the graphs of $g(x)$ and of $h(x)$ together on the same axes for various ranges of x-values *in the first quadrant* until you have one that clearly shows the regions bounded by these two graphs for $x \geq 0$.

In[167]:=
```
Plot[{p[x], q[x]}, {x, 0, 1.5}]
```

Out[167]=



- Get the x-values of that intersection point.

  Intersection point: $x \approx$ 1.17456

In[170]:=
```
NSolve[p[x] == q[x], x, Reals]
```

Out[170]= {{x → 1.17456}}

- Find the volume for the solid obtained by rotating about the x-axis the region bounded by $y = p(x)$ and $y = q(x)$ for $x \geq 0$.

  Volume: 23.049

In[173]:=
```
Integrate[Pi (q[x]^2 - p[x]^2), {x, 0, 1.1745594102929802`}]
```

Out[173]= 23.049