Lab 6

$\cos^2\theta + \sin^2\theta = 1$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \tan^2\theta = \sec^2\theta$

$$I = \int \frac{1}{\sqrt{16+x^2}}\,dx$$

$x = 4\tan\theta$

$dx = 4\sec^2\theta\,d\theta$

$$= \int \frac{4\sec^2\theta\,d\theta}{\sqrt{16+16\tan^2\theta}} = \int \frac{4\sec^2\theta\,d\theta}{4\sqrt{1+\tan^2\theta}}$$

$$= \int \sec(\theta)\,d\theta = \int \sec\theta\left(\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right)d\theta$$

$$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta+\tan\theta}\,d\theta$$

$$= \int \frac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta+\tan\theta}\,d\theta$$

$$= \int \frac{du}{u}$$

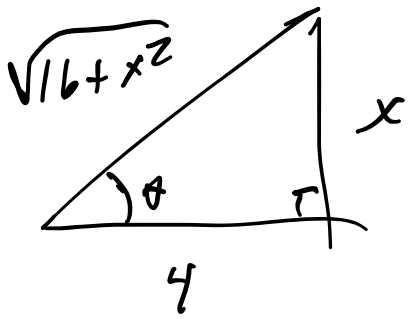
$u = \sec\theta + \tan\theta$

$du = (\sec\theta\tan\theta + \sec^2\theta)\,d\theta$

$$= \ln|u| + C$$

Note $|u|$, to make the anti-derivative even (since $\frac{1}{u}$ is odd).

$$= \ln|\sec(\theta)+\tan(\theta)| + C$$



$x = 4\tan\theta$

$\tan\theta = x/4$

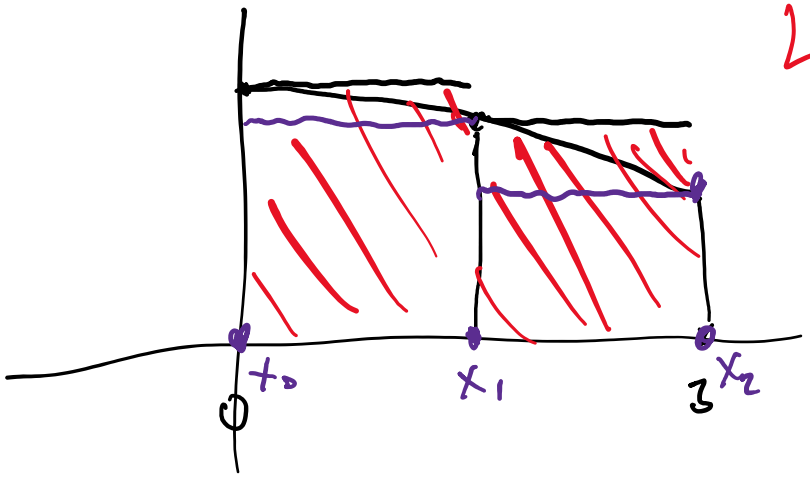
$\sec\theta = \frac{\sqrt{16+x^2}}{4} = \frac{hyp}{adj}$

$$= \ln\left|\frac{\sqrt{16+x^2}}{4} + \frac{x}{4}\right| + C$$

$$\int_0^3 \frac{1}{\sqrt{16+x^2}}\,dx = \ln\left|\frac{\sqrt{25}}{4} + \frac{3}{4}\right| - \ln\left|\frac{4}{4} + 0\right|$$

$$= \ln|2| - \ln|1|$$

$$= \boxed{\ln(2)}$$



Left is an overestimate

Right is an underestimate

$n = 2$