# Lab 7: Instructors' notes

Week 7, February 22

MAT 229, Spring 2021

## Techniques of integration

- Substitution
  - Calculus 1 substitutions: $\int f(u(x))\, u'(x)\, dx = \int f(u)\, du$
  - Trigonometric integrals: $\int \sin^m(x) \cos^n(x)\, dx$, $\int \tan^m(x) \sec^n(x)\, dx$, $\int \cot^m(x) \csc^n(x)\, dx$
  - Trigonometric substitutions
    - roots of $a^2 - x^2 \longrightarrow x = a \sin(\theta),\ dx = a \cos(\theta)\, d\theta$
    - roots of $a^2 + x^2 \longrightarrow x = a \tan(\theta),\ dx = a \sec^2(\theta)\, d\theta$
    - roots of $x^2 - a^2 \longrightarrow x = a \sec(\theta),\ dx = a \sec(\theta) \tan(\theta)\, d\theta$
- Integration by parts $\int u\, dv = u\, v - \int v\, du$ or $\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

## Numerical Integration Code: thanks to Al Hibbard -- HibbardA@central.edu

### Choose a technique

For each of the following integrals, decide which technique is appropriate and begin the process. Go until you think it will be successful.

**1.** $\int x\, e^{-x^2}\, dx$

**2.** $\int x\, e^{-x}\, dx$

**3.** $\int \sin^2(x) \cos^3(x)\, dx$

**4.** $\int \sec^2(x) \tan^3(x)\, dx$

**5.** $\int \sqrt{16 - x^2}\, dx$

**6.** $\int x \sqrt{16 - x^2}\, dx$

**7.** $\int \frac{\ln(x)}{x}\, dx$

**8.** $\int x \ln(x)\, dx$

## Numerical integration error analysis

Consider the definite integral $\int_0^4 \cos\left(\sqrt{x^3+1}\right) dx$.

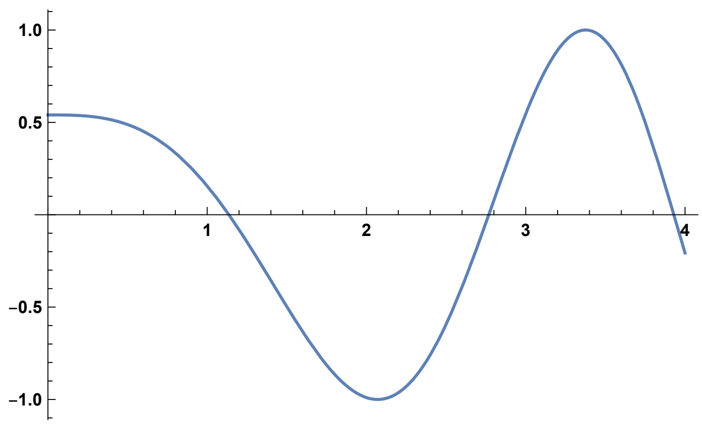
1. Using the trapezoid rule error estimate, determine a value of $n$ to use in the trapezoid rule to get error less than 0.0001.
2. Using the midpoint rule error estimate, determine a value of $n$ to use in the midpoint rule to get error less than 0.0001.
3. Using the Simpson's rule error estimate, determine a value of $n$ to use in Simpson's method to get error less than 0.0001.
4. Using whichever of the three above rules you prefer estimate $\int_0^4 \cos\left(\sqrt{x^3+1}\right) dx$ using the $n$-value you found for that method.

In[829]:=
```
f[x_] := Cos[Sqrt[x^3 + 1]]
true = NIntegrate[f[x], {x, 0, 4}]
```

Out[830]=
```
0.173354
```

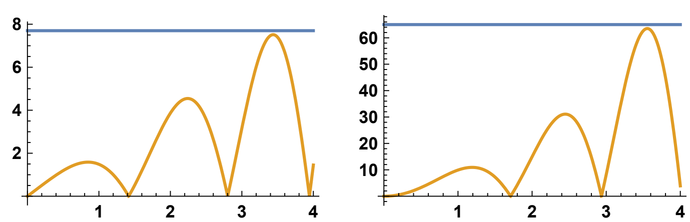
In[769]:=
```
Plot[f[x], {x, 0, 4}]
```

Out[769]=




In[777]:=
```
GraphicsGrid[
 {{Plot[{7.7, Abs[f''[x]]}, {x, 0, 4}], Plot[{65, Abs[f''''[x]]}, {x, 0, 4}]}}]
```

Out[777]=



In[815]:=
```
a = 0.0;
b = 4.0;
k2 = 7.7;
k4 = 65;
bound = 0.0001;
Clear[n]
midN = Ceiling[Max[n /. Solve[k2 (b - a) ^3 / (24 n^2) == bound, n]]]
Clear[n]
trapN = Ceiling[Max[n /. Solve[k2 (b - a) ^3 / (12 n^2) == bound, n]]]
Clear[n]
simpN = Ceiling[Max[n /. Solve[k4 (b - a) ^5 / (180 n^4) == bound, n, Reals]]]
```

Out[821]= 454

Out[823]= 641

Out[825]= 44

In[831]:=
```
mid = NMidpointApprox[f[x], {x, a, b}, midN]
trap = NTrapezoidApprox[f[x], {x, a, b}, trapN]
simp = NSimpsonApprox[f[x], {x, a, b}, simpN]
```

Out[831]= 0.173363

Out[832]= 0.173344

Out[833]= 0.173365

In[834]:=
```
errors = Abs[{mid, trap, simp} - true]
```

Out[834]= $\{9.42048 \times 10^{-6}, 9.4513 \times 10^{-6}, 0.0000111394\}$