Section Summary: 7.2: Trigonometric Integrals

a. **Definitions** None to speak of, although you should recall the definitions of the trig functions (in terms of sin and cos).

b. **Theorems**

Trigonometric identities are essentially theorems with well-known proofs. Among the most important are the following:

- $$\sin^2(x) + \cos^2(x) = 1$$

  (Pythagorean theorem). If we divide this by either $\sin^2(x)$ or $\cos^2(x)$, we get a new identity involving tan and sec, or cot and csc.

  With two more identities, we can derive every other identity mentioned here (or in this section):

  $$\sin(A + B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

  and

  $$\cos(A + B) = \cos(A)\cos(B) - \sin(B)\sin(A)$$

  For example, we get this identity from the "sine of a sum" formula, setting $A = B = x$:

- $$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$$

  (double angle formula). We can derive a formula for $\cos(2x)$ by using the "cosine of a sum" formula. (What is it?)

- $$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

  (half-angle formula), from which we can derive a formula for $\cos^2(x)$ by using the Pythagorean identity:

  $$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

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$$\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$$
$$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$
$$\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$$

  You might wonder what's so exciting about these types of integrals: it turns out that they occur in an essential way in an important technique called Fourier analysis. The idea is that we can approximate relatively arbitrary functions in terms of trig functions, but we won't go into that now.

c. Properties/Tricks/Hints/Etc.

d. Summary

This section might be considered "archaic" by some: it's full of techniques which we really don't need much anymore, since, if we're confronted by some hairy integral of this class we would be able to get the solution most easily by asking your TI calculator or Mathematica to solve it...!

One of the important lessons of this section is the usefulness of trigonometric identities, many of which you've undoubtably forgotten (if you ever were asked to learn them). Take this opportunity to remember the relationship between tan, sec, sin, and cos, etc., and their derivatives.