Weekly Assignment 8

Instructions: **Don' t leave these to the last minute -- they aren't a cake walk!**
Feel free to reach out during office hours for hints and help!:)

1. Comparison test

For each of the given series, determine if it converges or not. Give reasons for your answer. If it converges, approximate it with error less than 0.0001.

a. $\sum_{k=0}^{\infty} \frac{3^k}{k^4+4^k}$

$\frac{3^k}{k^4+4^k} < \frac{3^k}{4^k} = \left(\frac{3}{4}\right)^k$ for all k

```
In[608]:= Clear[n]
          soln = Solve[(3/4)^(n+1)/(1-3/4) == .0001]
          enn = n /. soln[[1]]
          enn = Ceiling[enn]
          N[Sum[3^k/(k^4+4^k), {k, 0, enn}]]
```

approx=

```
Out[609]= {{n → 35.8345}}
Out[610]= 35.8345
Out[611]= 36
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$|S - S_n| \le \sum_{k=n+1}^{\infty} \left(\frac{3}{4}\right)^k < 0.0001$

find n

Out[612]= $\boxed{3.01655}$

b. $\sum_{k=0}^{\infty} \frac{5k^2+k+1}{2k^3+k^2+10}$ > $\frac{5k^2}{2k^3}$ (eventually)

This one is diverging, like 1/k.

c. $\sum_{k=0}^{\infty} \frac{\sqrt{9k^2-k}}{6k^4+7}$

converges like $1/k^3$

$\frac{\sqrt{9k^2-k}}{6k^4+7} < \frac{3k}{6k^4} \sim \frac{1}{2k^3}$

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In[613]:= Clear[n]
          soln = Solve[Integrate[1/2 × 1/x^3, {x, n, Infinity}] == .0001, n]
          enn = n /. soln[[1]]
          enn = Ceiling[enn]
          N[Sum[Sqrt[9 k^2 - k]/(6 k^4 + 7), {k, 0, enn}]]
```

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Out[614]= {{n → 50.}}
Out[615]= 50.
Out[616]= 50
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Out[617]= $\boxed{0.311754}$

2. Alternating series

a. Consider the series $\frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \ldots$. This is a convergent series by the alternating series test. Using an appropriate error estimate determine how many terms are needed of this series to approximate the infinite sum with error less than 0.0001.

1st job - construct the formula for the series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \underbrace{\frac{4}{2k-1}}_{b_k}$$

1st neglected term

```
In[618]:= Clear[n]
          soln = Solve[4 / (2 (n + 1) - 1) == .0001, n]
          enn = n /. soln[[1]]
          enn = Ceiling[enn]
          N[Sum[(-1) ^ (k + 1) 4 / (2 k - 1), {k, 1, enn}]]
Out[619]= {{n → 19 999.5}}
Out[620]= 19 999.5
Out[621]= 20 000
Out[622]= 3.14154
```

b. Another series that converges to the exact same value as the series above is $\sum_{k=0}^{\infty} \frac{(-1)^k 2}{(2k+1) 3^{k-1/2}}$. Using an appropriate error estimate determine how many terms are needed of this series to approximate the infinite sum with error less than 0.0001.

1st neglected term

always round up

```
In[623]:= Clear[n]
          soln = Solve[2 / ((2 (n + 1) + 1) 3^ (n + 1 / 2)) == .0001, n]
          enn = n /. soln[[1]]
          enn = Ceiling[enn]
          N[Sum[(-1) ^k 2 / ((2 k + 1) 3^ (k - 1 / 2)), {k, 0, enn}]]
Out[624]= {{n → 6.04422}}
Out[625]= 6.04422
Out[626]= 7
Out[627]= 3.14157
```

3. Choose either of the above two series and approximate it with error less than 0.0001. What famous number do these two series series converge to?

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In[ ]:= N[Pi, 52]
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